



SELINUS UNIVERSITY
OF SCIENCES AND LITERATURE

**A Cliodynamics Approach to the Temporal and
Spatial Distribution of Early Medieval
Kingdoms**

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“I do hereby attest that I am the sole author of this project/thesis and that its contents are only the result of the readings and research I have done”.

Abstract

This research examines England's early medieval period or the Dark Ages. Temporally, this period follows the Roman empire's collapse in 410 CE and extends until the end of the Anglo-Saxon period in 1066 CE with the Norman conquest. Spatially, it covers the geographical area of Britain south of Hadrian's Wall. Mathematical and computational techniques derived from the discipline of Cliodynamics are used to model the processes occurring during the early medieval age in England.

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Chapter 1 Introduction

After the Roman Empire's end, Britannia's Roman province was fragmented into many small communities scattered across the British Isles. These small communities, over a period of centuries, gradually merged into larger communities following a mainly rural, agrarian lifestyle. From approximately 450 CE, Germanic tribes customarily known as Saxons, Jutes, and Angles settled in Britain. These tribes gradually displaced the Britons and underwent a series of mergers and fragmentations characterised by warfare between them. This led to the formation of small kingdoms commonly called the Heptarchy (Yorke 1990).

Later in this period, from approximately 780 CE, the Vikings raided and eventually invaded and occupied the northern portion of England. The Saxon Kingdom of Wessex merged with the surviving Anglo-Saxon Kingdoms, and essentially, England was formed. This is a very complex scenario, but there is a variety of documentary, historical and archaeological evidence of this period, which provides a basis for discovering the distribution of the population of England during the early medieval period. (Higham & Ryan, 2013)

In this research, mathematical and computational techniques derived from the discipline of Cliodynamics are used to model the processes occurring during the early medieval age in England (Turchin 2003a). The techniques include using the Lotka-Volterra equations to model the evolution of competitive populations over time (Turchin 2003b). Insights from geopolitical theory are also used to inform the modelling process, particularly of the variables influencing the coefficients of the Lotka-Volterra equations (Collins & Sanderson 2009). The spatial modelling approach of Artzrouni and Komlos (1996) is adapted within Cliodynamic computational simulations to model how warfare affects societies (Turchin, 2011) and (Turchin et al 2013).

The methodology used employs primary and secondary sources and computational modelling. The primary documentary materials are held by institutions such as the British Library and the National Archives, and the archaeological reports are held by English Heritage and the Archaeological Data Service (ADS). There is a wide variety of secondary sources, including journals, books and conference proceedings covering aspects of the topic, including geographic information system (GIS) databases.

The computational models are implemented in Python (Van Rossum, 2009) using various packages such as Pandas (McKinney, 2010), Geopandas (Jordahl, 2014), NumPy, and SciPy (Virtanen, 2020), supported by an open-source GIS package called QGIS. The outputs from this process will be maps of simulated early medieval kingdoms in the form of shapefiles and data tables.

The research outcome is primarily a series of computational models that can simulate the processes of state formation and fragmentation in the context of Early Medieval England. This is valuable since Cliodynamics has not yet been applied in this context, so the research expands the range of societies studied in this field. It is also useful in that it increases confidence in the findings reported when Cliodynamics is applied in other situations such as societal collapse (Brozović 2022), cyclical events (Turchin & Nefedov 2009) and responding to environmental change (Hussain et al 2020).

Historical Background

The early medieval world emerged from the Anglo-Saxon migrations of the 5th and 6th Centuries into the remnants of Roman Britain, from which the Romans had withdrawn around 400 CE. This was a time of major transition in Britain and elsewhere in Europe. Near-contemporary documentary sources survive, namely Gildas' writings from the 6th Century (Gildas, 2010), which describe the history of Britain before and during the migrations. Later texts include the Venerable Bede's 8th-Century 'Ecclesiastical History of the English People' (Bede, 1990) and the Anglo-Saxon Chronicles (Carruthers and Ingram, 2013). Politically, Britain came to be organised into a handful of larger kingdoms. The rise of kingship from the 7th Century was increasingly reflected in settlement forms and mortuary practices. Coins were produced by Anglo-Saxon rulers from this point. Trading centres known as wics, and emporia were also established at this.

Christianity was not officially introduced until 597 CE when St. Augustine's mission in Kent was first recorded. Its subsequent spread was aided by royal patronage (Blair 2005). It was not until the late 8th Century that the Scandinavian attacks began and were recorded in the various chronicles of Britain. These persistent incursions were soon followed by the long-term settlement of the invaders, establishing various towns such as York in England and Dublin in Ireland.

Archaeological Background

The early medieval period in Britain has been relatively well explored by archaeologists; traditionally, these studies have focused on relatively high-status settlements and ecclesiastic centres. In recent decades, archaeological work has especially been seen before infrastructural and other developments. This has led to the availability of unprecedented numbers of

excavation reports, large numbers of associated radiocarbon dates, and environmental archaeological evidence from all periods. Much more data is now available than before, and crucially, these datasets have been collected with much less influence from the research bias that can strongly affect the patterning of archaeological data. This is because the work is dictated by the pressures of development rather than previous surveys or the presence of above-ground-level indicators such as megalithic tombs, castles, etc.; many thousands of sites have been discovered simply by maintaining a watching brief on the topsoil stripping in advance of building work or road construction.

Although the deluge of data generated by this activity is still being processed, the overall trends emerging from the work have already become apparent. There is a growing realisation that past activity levels were highly dynamic, contrasting traditional growth models. Such models viewed the past as gradual increases in social and technological complexity, punctuated with revolutions such as the Neolithic introduction of agriculture or the Industrial Revolution of the 19th Century. By contrast, the highly resolved discoveries of archaeological work in recent decades reveal episodes of expansion and recession. Therefore, some time periods are associated with many more sites than others. Within the eight-century span of early medieval England, fluctuating activity levels have been detected in various aspects of its archaeology.

In summary, archaeological evidence from early medieval England points to fluctuating activity and economic change. Recent discoveries of many settlement sites present the opportunity for quantitative data analysis using newly developed methods.

Cliodynamics background

Cliodynamics is an interdisciplinary field that uses quantitative tools and historical data to develop and test general principles of history. The mathematical techniques used are like those used in computational modelling ecosystems, urban dynamics, and artificial societies. These techniques include differential equations like the Lotka-Volterra models, power-law relations like Zipf's law, and agent-based models comparable to the Schelling model. Other widely used techniques include evolutionary game theory, social network analysis, and systems dynamics techniques. Simulation techniques balance the deterministic nature of the general historical principles and the random or stochastic aspects of historical events. Because of this, statistical analysis plays an important role in testing the hypotheses that have been developed. Cliodynamics are often applied to medieval agrarian societies, such as France and Russia (Turchin, 2009), because they are less complex than later industrial societies, and their historical dynamics are similarly less complex. However, as the discipline develops, modelling more complex contemporary societies is being undertaken more frequently (Turchin, 2017).

Chapter 2 Literature Review

Publicly available datasets have been crucial to this research. In Britain, the Archaeological Data Service (ADS), based in York, collates many of these. The ADS website acts as a portal to thousands of archaeological datasets and reports.

The distribution of English archaeology has been extensively researched through the English Landscape and Identities (EngLaID) project (Green and Creswell, 2021). This project collected extensive data from various sources, including English Heritage's National Mapping Program (NMP), Historic Environment Records (HERs), and the Portable Antiquities Scheme (PAS). Integrating these diverse datasets posed significant challenges due to differences in spatial and categorical precision, requiring the use of advanced technologies such as semantic web, linked data, and GIS.

The project also found changes in agricultural and ritual practices during the middle/late Anglo-Saxon period, indicating a persistent link between agricultural activities and religious practices. Additionally, variations in archaeological evidence density were linked to factors such as modern land use, past material culture usage, and population density, which sometimes obscured genuine historical patterns.

To address biases in archaeological distributions caused by modern land-use changes, point pattern analysis (PPA) and custom bias surfaces were used to enhance the understanding of site distributions and their environmental preferences. Overall, the EngLaID project demonstrates the integration of large-scale spatial datasets and advanced analytical methods to uncover the complex spatial dynamics of English archaeology.

An atlas of Anglo-Saxon England (Hill, 1981) is invaluable in understanding early medieval England's spatial dynamics. It provides a graphical summary, including many maps and charts, covering various geographical aspects such as sea-level changes, settlement patterns, invasions, royal itineraries, land holdings, mints, and coinage. The text *English Landscapes and Identities* (Gosden, 2021) presents a similar range of topics, incorporating materials from recent decades of archaeological investigation, including the Portable Antiquities Scheme (PAS) project (Naylor, 2020) and The Viking and Anglo-Saxon Landscape and Economy (VASLE) project (Richards, 2009). The text, *The Shaping of the English Landscape* (Green and Creswell, 2021), presents a visually stunning overview of England's landscape archaeology. Works on Tribal Hideage (Blair, 1995), the Burghal Hideage (Haslam, 2016) and the Hundreds of the Domesday books (Brookes, 2020), (Reyolds, 2019) give an impression of the extent of Anglo-Saxon territories and their administrative boundaries.

In the work by Scull (2020), a critical re-examination of the social, spatial, and temporal dynamics of great hall complexes in England from the 6th to the 8th century is presented. The research delves into critical interpretative challenges and limitations the data poses and subsequently scrutinises them through a comparative analysis spanning both long-term and short-term temporal scales. Gretzinger (2022) explores Anglo-Saxon migration as part of a continuous movement of people across the North Sea to Britain from the later Roman period into the eleventh century CE. Ormond (2020) provides an in-depth analysis of migration in medieval England, exploring the movement of people and the social and cultural effects of migration. Work by Cooper and Green (2017) discusses the interpretation of spatial and temporal data from large datasets such as the PAS. A significant genetic study (Leslie, 2015) has provided a deeper insight into the migration period of early medieval English history.

This thesis interprets archaeological and historical data using techniques derived from the discipline of Cliodynamics. Turchin and his collaborators developed the discipline's theoretical basis in a series of texts, including *Historical dynamics* (Turchin, 2003b), *War and peace and war* (Turchin, 2006), *Secular Cycles* (Turchin and Nefedov, 2009), *Ultrasociety* (Turchin, 2016), and *Ages of Discord* (Turchin, 2016). In the text *Complex Population Dynamics* (Turchin, 2003a), much of the mathematical apparatus of Cliodynamics appears in the context of ecosystems and their dynamics.

Chapter 3 Data and Methodology

Cliodynamics is based on the idea that history can be studied using quantitative tools and historical data to test theories and make predictions (Turchin, 2011b). It assumes that there are general social mechanisms and laws of history which can be identified through empirical patterns and scientific predictions. By examining various empirical regularities in historical social systems, the underlying principles driving these patterns can be identified. Mathematical models can then be used to predict linked population-instability oscillations, highlighting the interconnected nature of population dynamics and political instability.

Cliodynamics focuses on four main variables to analyse long-term social trends: population numbers, social structure, state strength, and political instability. Each variable is measured in several ways using quantitative proxy data. Choosing appropriate proxies can be a challenge because relevant data are often hard to find and since no proxy is perfect, several proxies are used for each variable to minimise the problem (Spinney, 2012). This data can be obtained from a variety of sources, including databases such as:

- Glass Beads from Anglo-Saxon Graves
- Anglo-Saxon Kent Electronic Database (ASKED)
- Anglo-Saxon Graves and Grave Goods of the 6th and 7th Centuries AD: A Chronological Framework
- A corpus of Anglo-Saxon cruciform brooches
- Portable Antiquities Scheme Database
- Viking and Anglo-Saxon Landscape and Economy (VASLE) Project
- Unlocking Early Anglo-Saxon Girdle-Hangers
- Cloth and Clothing in Early Anglo-Saxon England, AD 450-700
- Geodatabase of Rural Anglo-Saxon Settlements
- Feeding Anglo-Saxon England (FeedSax)
- The Fields of Britannia project

The methodology adopted for these databases depends on the characteristics of the data. Generally, the dataset is imported into a Panda or geopandas dataframe depending on whether it contains location data. The temporal data is then extracted and binned using either Matplotlib

histograms (Hunter, 2007) and/or the Seaborn kdeplot (Waskom, 2021) to display the temporal structure.

The spatial structure of the location data is displayed using either Matplotlib scatterplot or Seaborn kdeplot with or without contours and a specific palette choice such as ‘spectral’ to enhance the visualisation. Spatiotemporal data is visualised using the time segmentation of location data displayed as PNG files in the main thesis or animated gifs in an appendix.

Archaeological Evidence

The English Landscape and Identities project

Many of the analyses and representations used in this thesis were inspired by the work of The English Landscape and Identities (EngLaId) project, which presents a set of density models for England showing the variation in the densities of archaeological evidence across wide time periods. Three possible explanations for the variation seen are discussed (Green, 2017), modern affordance, variability in past usage of material culture and past population density. These factors need to be borne in mind when temporal and spatial distributions are interpreted and accounted for if possible.

Radiocarbon dating

The remains of organic materials such as wood, bone, and ashes can be dated using C14 dating. By analysing the distribution of these remains across England by location and date, researchers have found a clear pattern of past human activity (Bevan, 2017). Contemporary archaeological studies use data-driven methods to analyse the frequency of dated archaeological materials over time to estimate levels of past human activity (e.g. Bevan et al. 2017; Brown 2017; Timpson et al. 2014; Williams 2012). This approach is a well-established but still somewhat problematic way to investigate large-scale changes in human populations and cultural sequences so summed radiocarbon dates are a good proxy for population (Contreras and Meadows, 2014).

Radiocarbon dates - which provide the ages of once-living biological samples - are particularly suitable for this analysis because they are commonly sought and widely published. They also come with contextual information and are numerical data since they consist of a measurable value and an associated error term, or when calibrated, an age range that defines the probable timeframe for a sample's age.

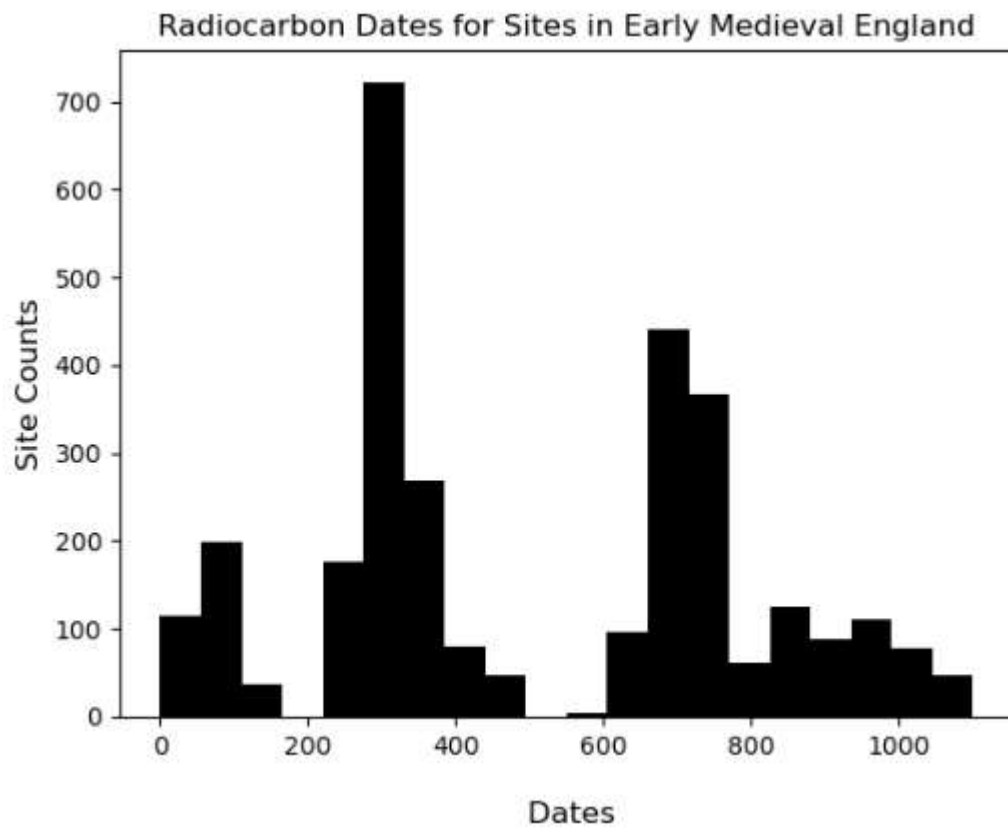


Figure 3-1 Temporal variation of Radiocarbon sites

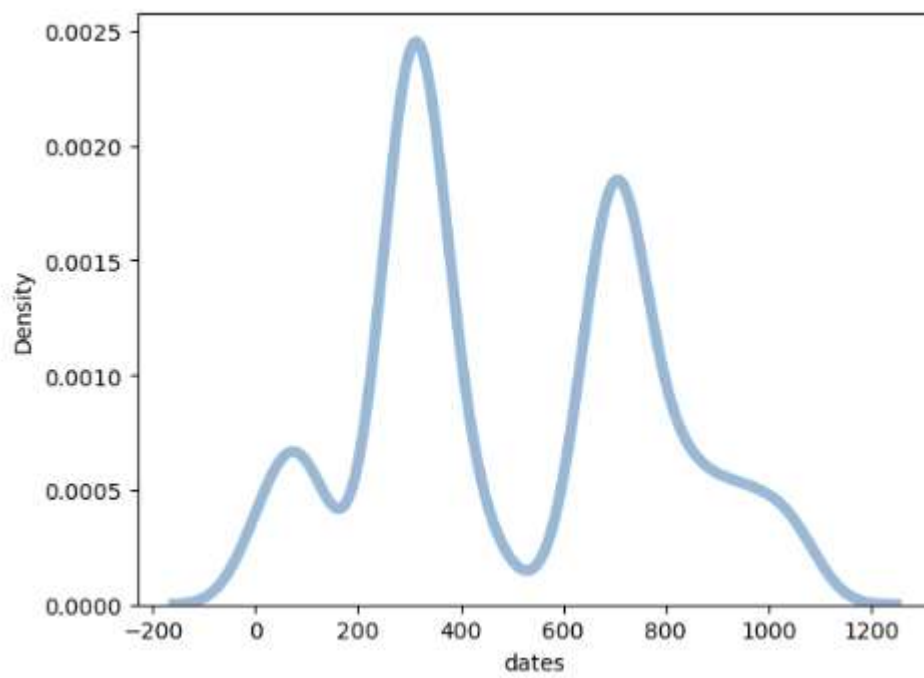
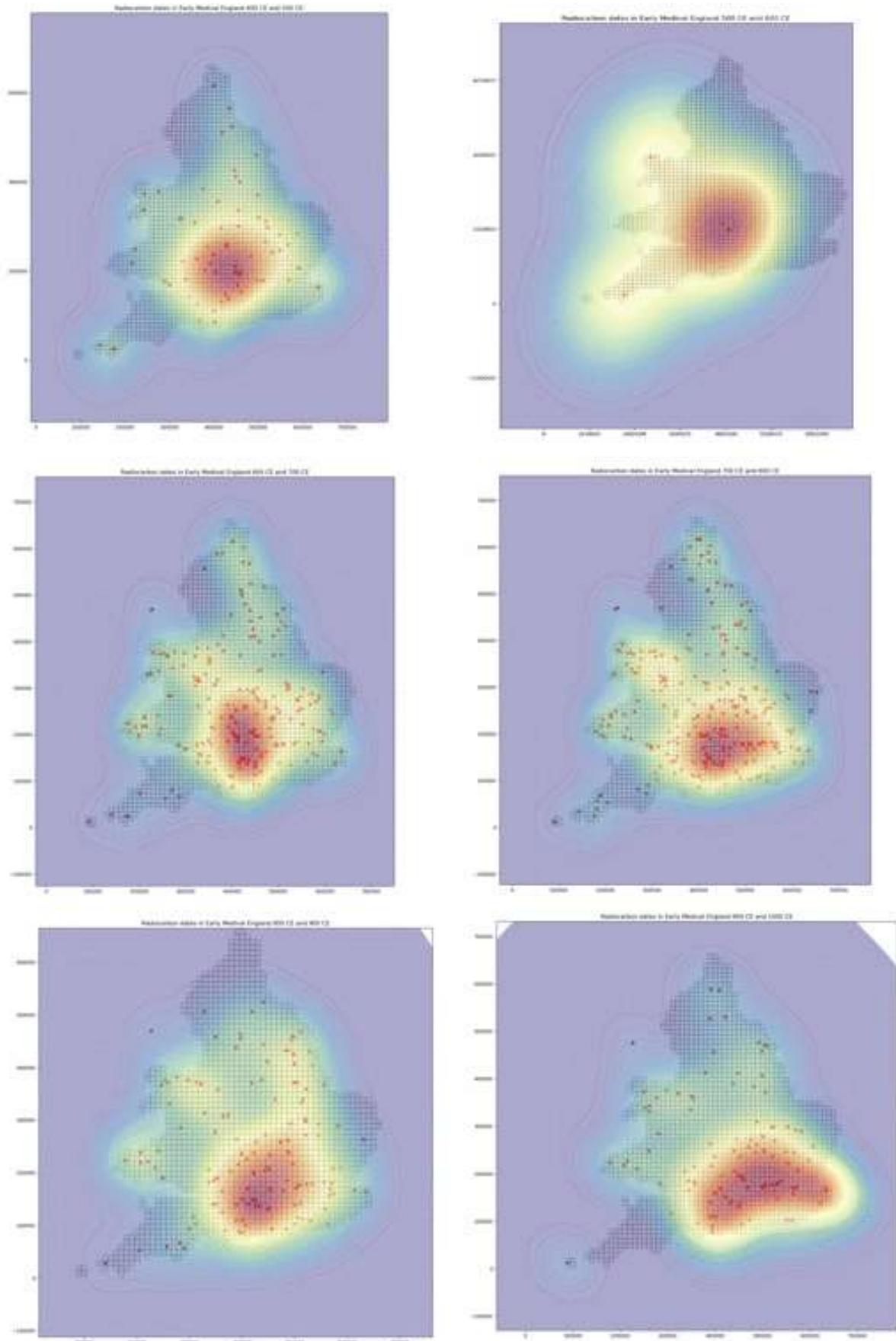


Figure 3-2 Kdeplot of Temporal Variation of Radiocarbon sites



Portable Antiquities Scheme

The Portable Antiquities Scheme (PAS) is a long-term project that catalogues all the archaeological finds made in Great Britain, including those found by metal detectorists. Using Pandas, Geopandas and Scikit, the complete PAS database of early medieval archaeological finds was analysed to investigate the spatial and temporal variation of activity. Additional Fast Fourier Transform was used to analyse the frequency distribution of temporal data.

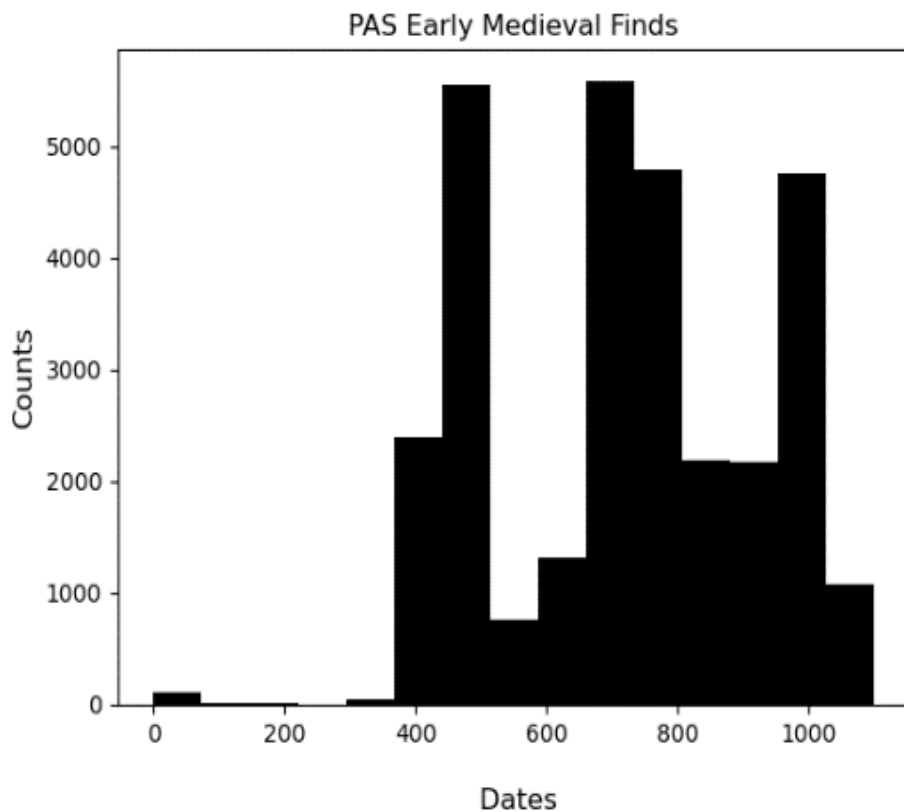


Figure 3-4 Temporal Variation of PAS Early Medieval Finds

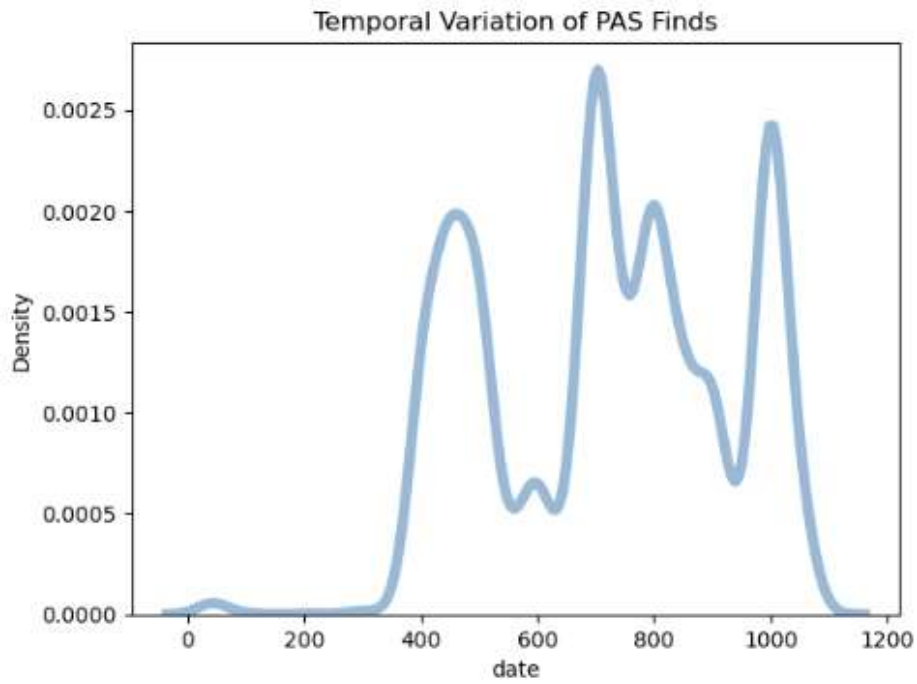


Figure 3-5 Kdeplot of Temporal Variation of PAS Early Medieval Finds.

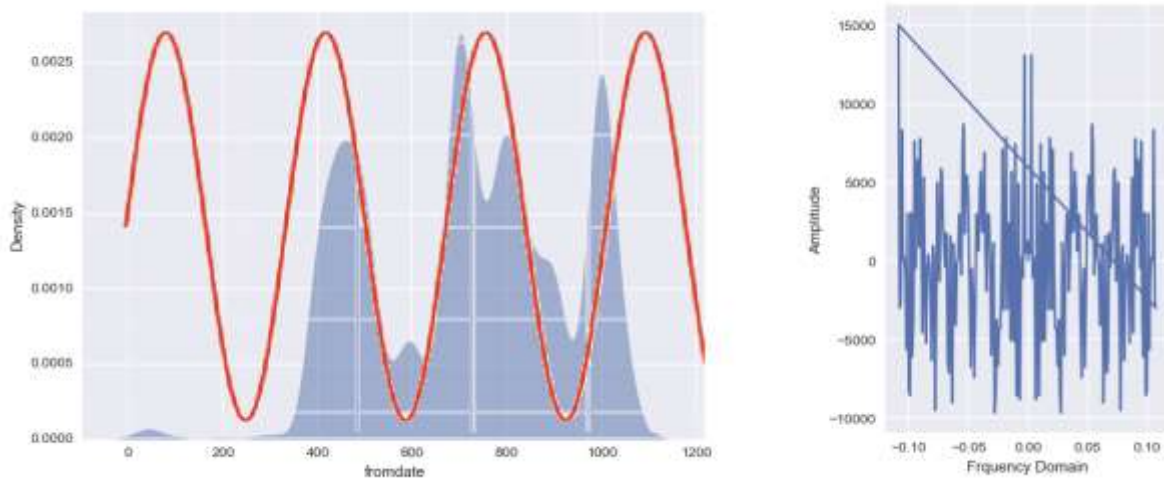


Figure 3-6 FFT analysis of PAS temporal variation

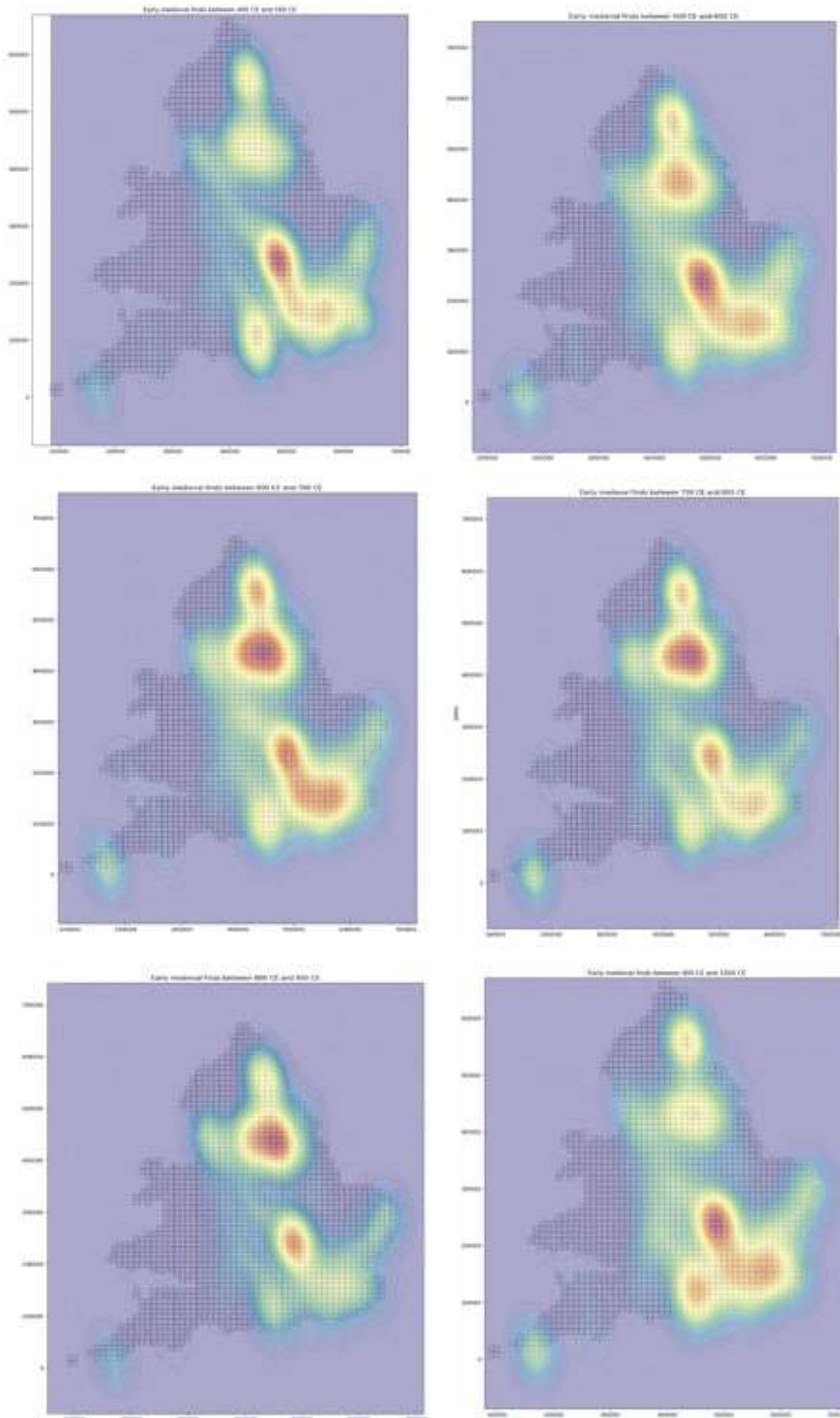


Figure 3-7 Spatial Variation of Early Medieval Finds vs Date

Early Medieval Coin Single Finds

A spatial and temporal analysis of coin finds is useful because coins can often be accurately dated. The British Numismatic Society (BNS) keeps databases such as the Corpus of Early Medieval Coin Finds (EMC) and collections of British coins, and it collaborates with the PAS.

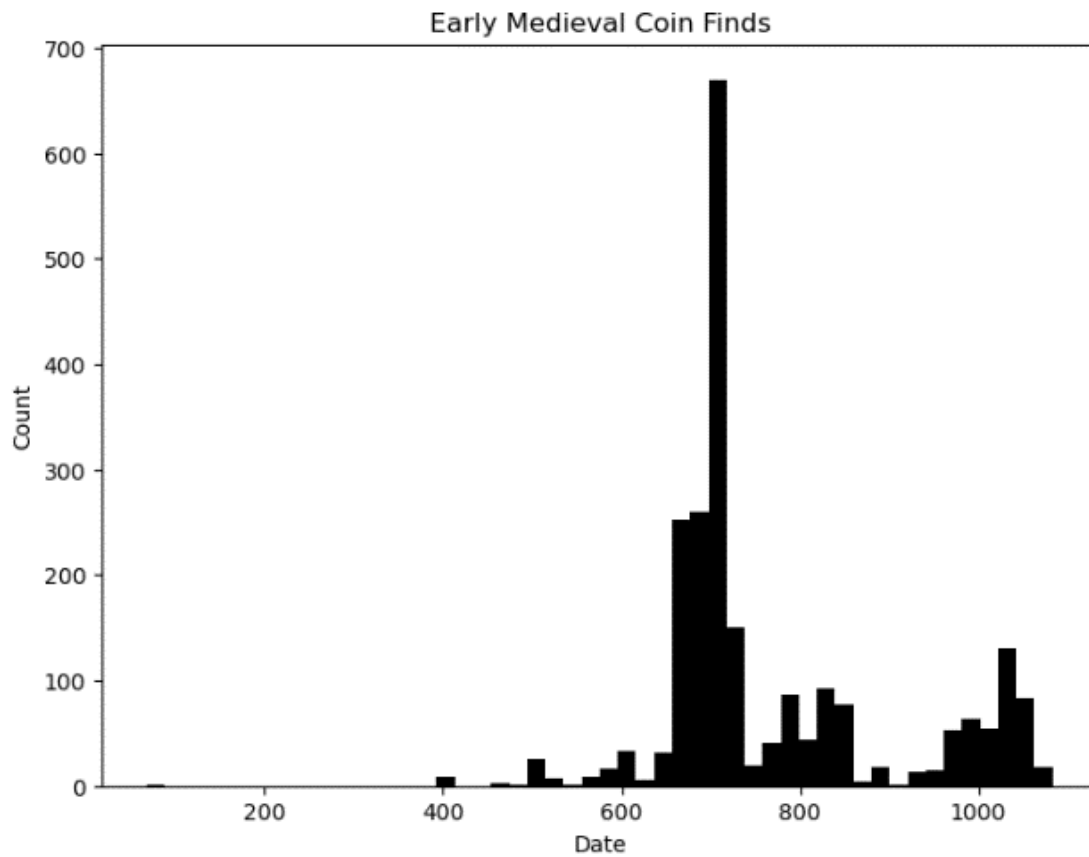


Figure 3-8 Temporal Variation of Early Medieval Single Coin Finds.

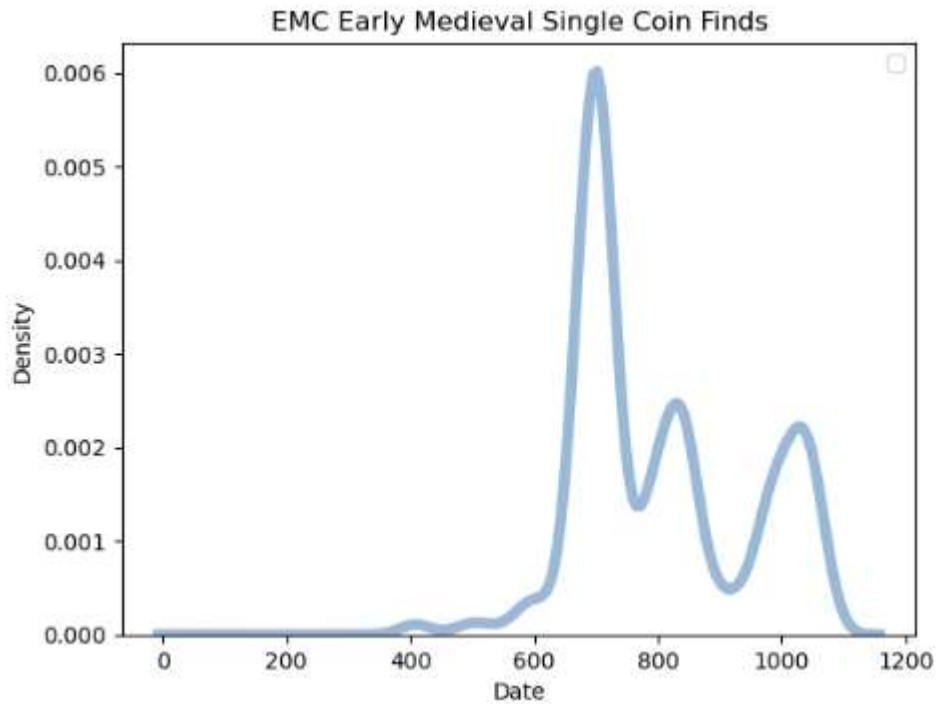


Figure 3-9 Kdeplot of Temporal Variation of EMC Early Medieval Single Coin Finds.

Single finds of coins are believed to represent random losses and are a very valuable window into the makeup of the currency of the time. Their interpretation needs caution however as not all finds are reported, and the local distribution is affected by modern land use and searching patterns as well as the actual distribution of medieval material (Naismith et al, 2014)

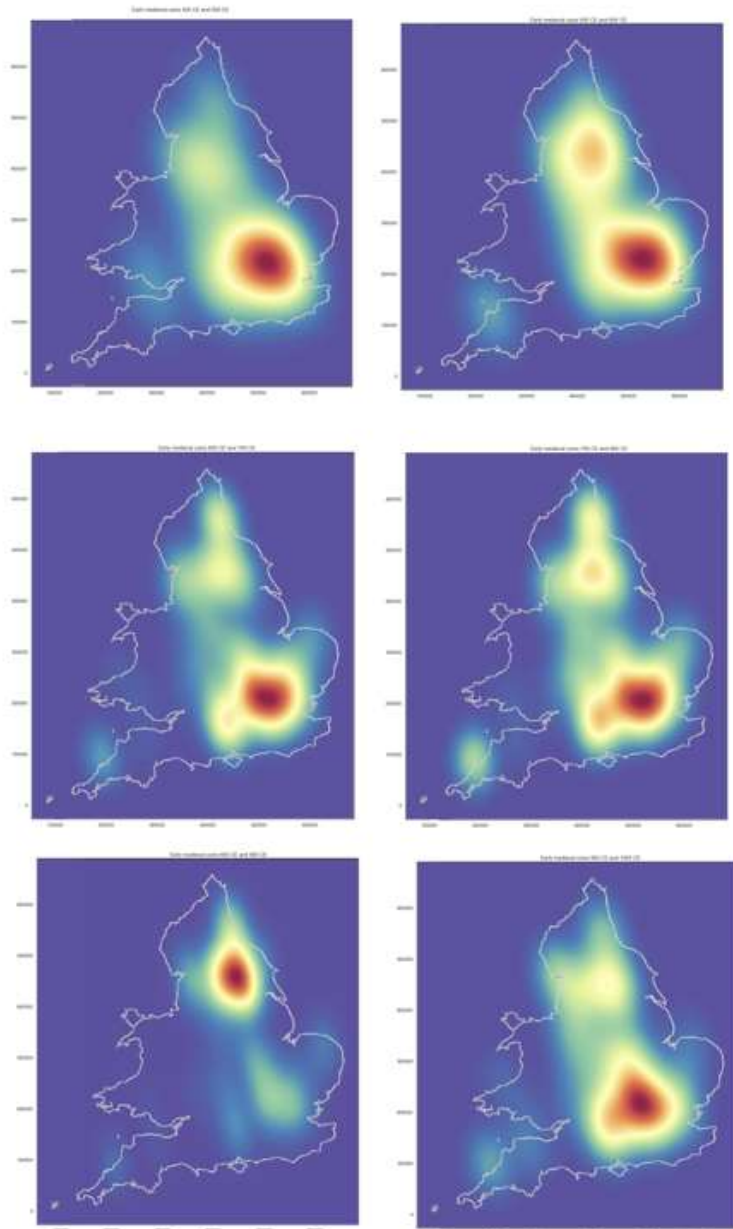


Figure 3-10 Spatial Distribution of Early Medieval Coin Finds

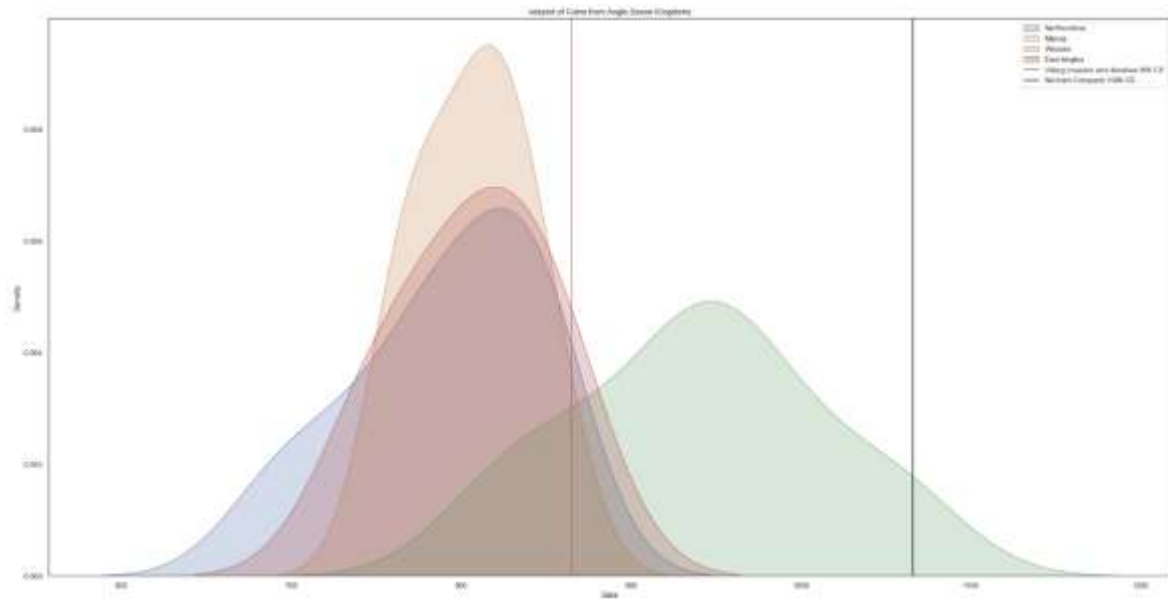


Figure 3-12 Temporal Distribution of Early Medieval Coins by Kingdom

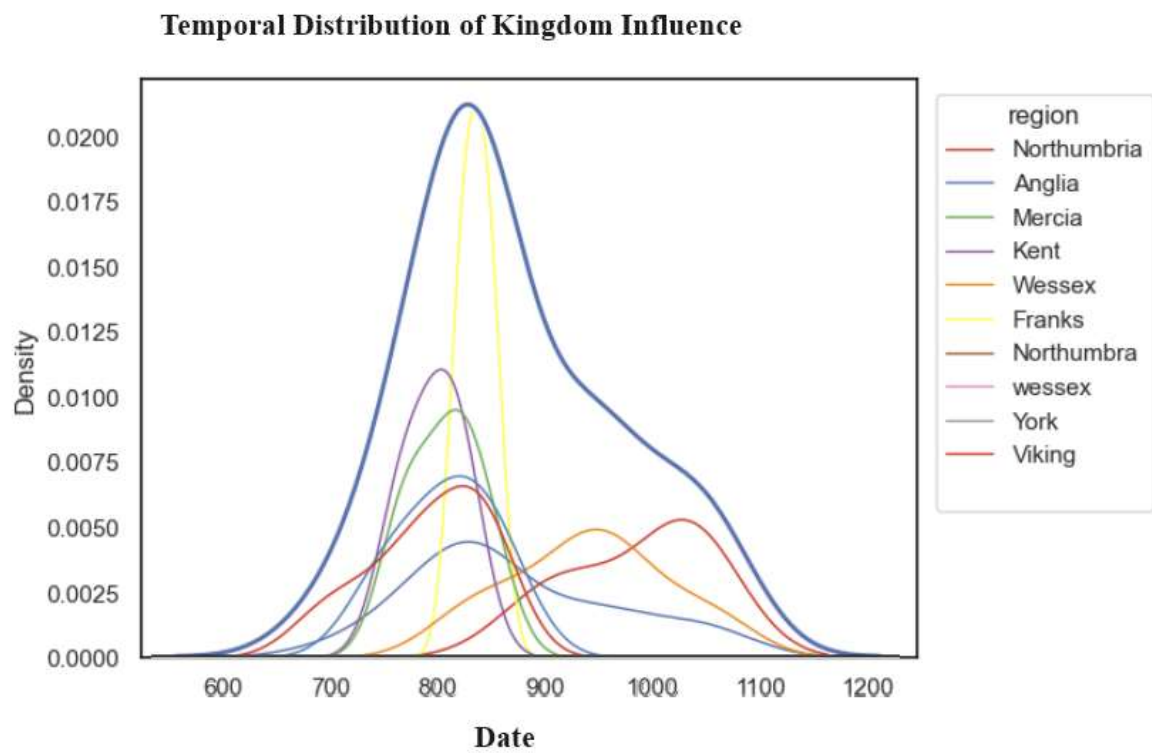


Figure 3-13 Anglo-Saxon Kingdom Influence over Time

example, hoards found in areas with fertile agricultural land or trading centres suggest these regions were prosperous. The locations of hoards can sometimes be linked to known historical events, such as battles or military campaigns. Hoards found along known invasion routes or near battle sites can offer insights into these events.

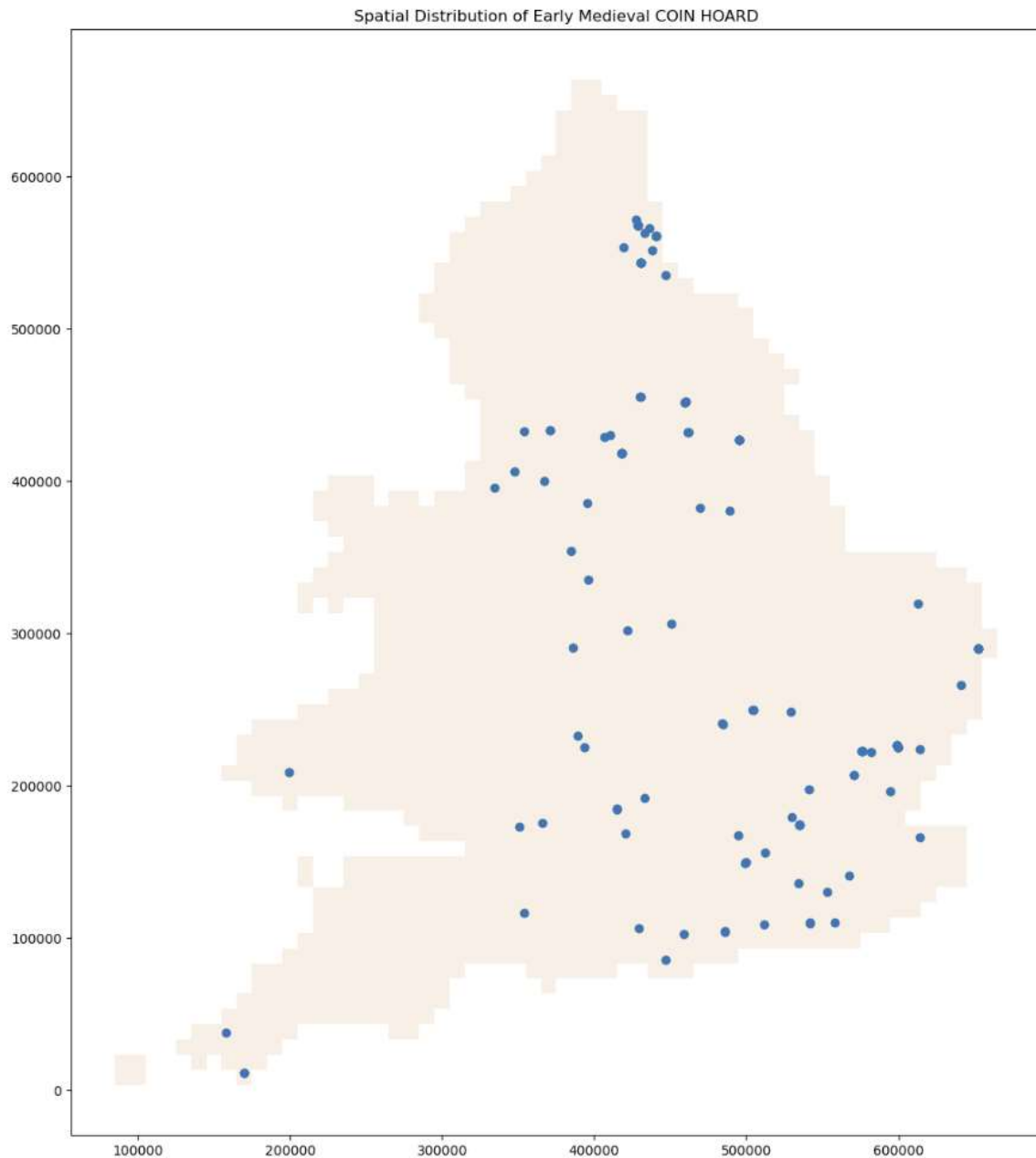


Figure 3-15 Spatial Distribution of Early Medieval Coin Hoards

The frequency and size of hoards may directly indicate economic fluctuations. However, Turchin and Scheidel (2009) propose that increases in hoarding occur during periods of political instability or with increasing threat of invasion, as people seek to safeguard their wealth. The frequency of coin hoards is used as a quantitative proxy for internal warfare, civil unrest, and instability.

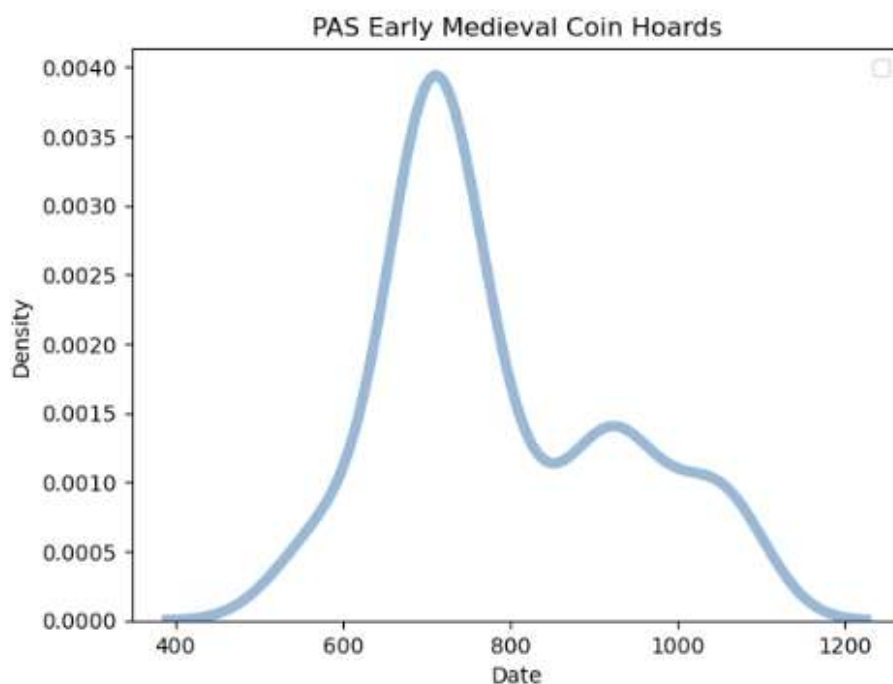


Figure 3-16 Kdeplot of Temporal Variation of PAS Early Medieval Coin Hoards

PAS Early Medieval Non-Coin Hoards

The distribution of non-coin hoards in different locations over time can help us understand settlement patterns and social structure in the early medieval period. Areas with a high concentration of non-coin hoards may indicate the presence of settlements and can also provide insights into whether these settlements were large central hubs or smaller dispersed communities. The specific objects within the hoards, such as intricate jewellery or weapons, can offer clues about the social status of the individuals or communities who buried them. Hoards containing tools or unfinished items may reveal centres of specialized craft production, shedding light on the economic activities of various regions. Moreover, the presence of exotic

materials or objects not native to England suggests trade connections with other regions, possibly even extending to long-distance networks.

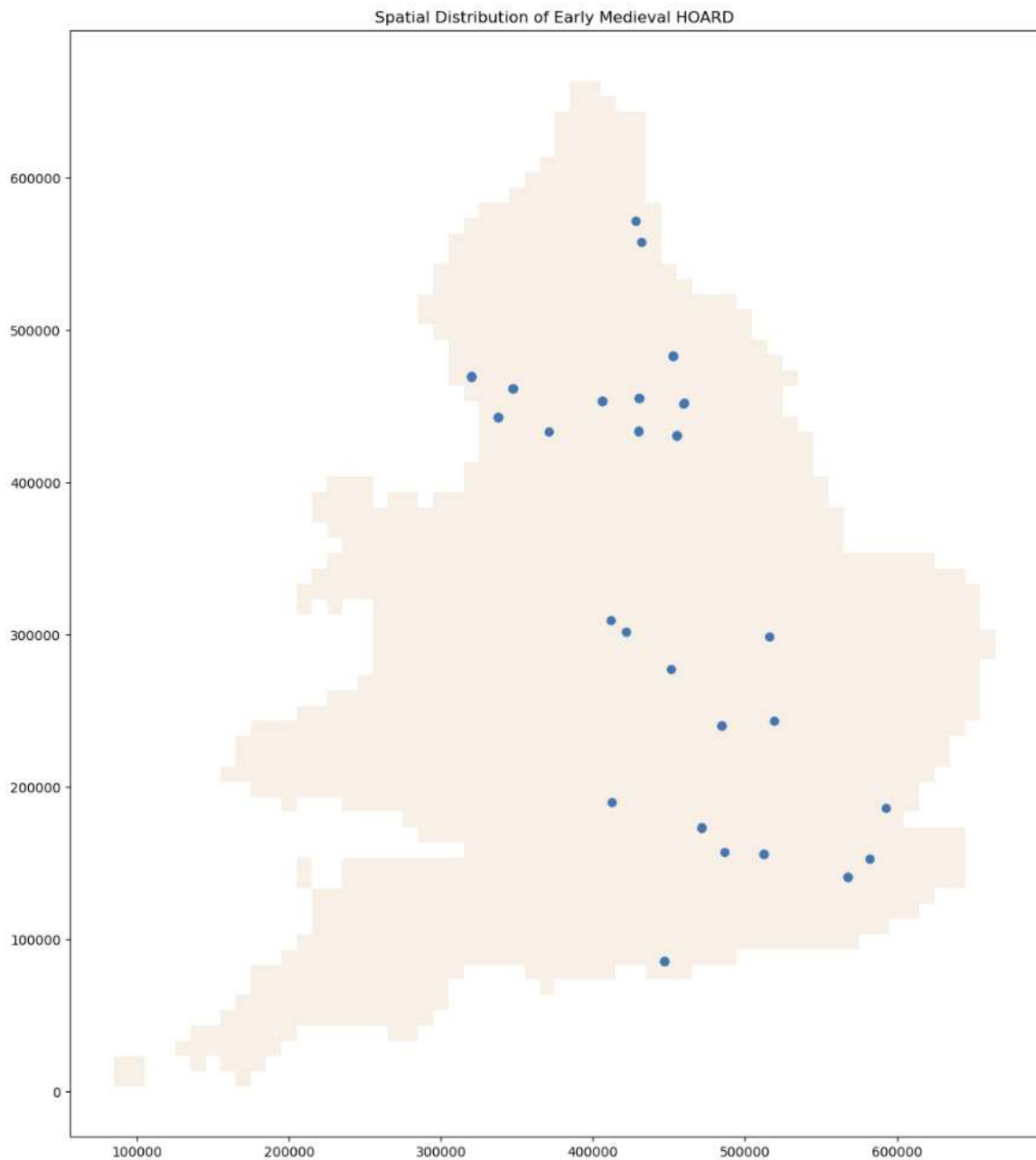


Figure 3-17 Spatial Distribution of non-coin hoards

The analysis of hoards containing weapons, particularly those found in strategic locations, can indicate periods of conflict and the types of weapons used. Hoards buried in haste or hidden in defensive structures could suggest periods of insecurity or imminent threat.

"By comparing the styles and types of objects within different hoards, it is possible to establish a relative chronology of the Anglo-Saxon period, even in the absence of absolute dates. In some cases, non-coin hoards can be linked to historical events or figures, providing a tangible connection between archaeological evidence and written records."

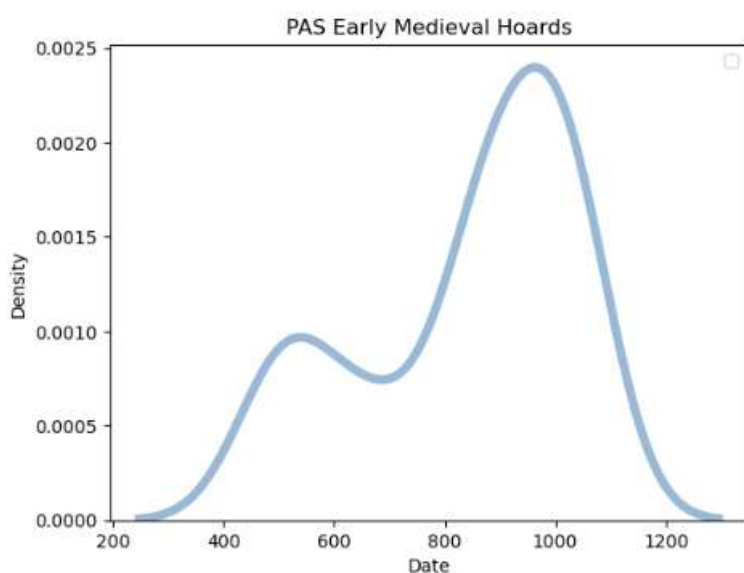


Figure 3-18 Temporal Distribution of Early Medieval non-coin Hoards

Coin hoards and non-coin hoards provide different but complementary insights into the early medieval period. Coin hoards are typically analysed for economic and monetary purposes, while non-coin hoards offer a broader view of the social, cultural, technological, and ritualistic aspects of the time. However, these two types of hoards are not mutually exclusive sources of information. Instead, they often work together to provide a more complete understanding of a particular site or community. For example, analysing a coin hoard in conjunction with non-coin hoards from the same site can offer a more comprehensive picture of the economic activities, social structure, and cultural practices of that community. By combining evidence

from both types of hoards, a more detailed and holistic understanding of the Anglo-Saxon period in England—including its economic, social, cultural, and technological dimensions—can be obtained.

Settlement data

Features of settlement patterns, such as an increase in the number and size of settlements like villages, towns or cities, can indicate population growth. A higher density of settlements in a region may suggest a larger population. Analysing the distribution of settlements can provide insights into population density patterns. Estimating the number of households within each settlement, based on archaeological evidence or historical records, can help calculate the overall population.

The Geodatabase of Rural Anglo-Saxon Settlements (Keil and Hamerow, 2014) and its accompanying text Hamerow, H. (2012) presents archaeological evidence of Anglo-Saxon rural settlements in England in a format suitable for spatial analysis and visualization. It includes details about the presence of archaeological features, site occupation dates, and selected bibliographic information for 84 sites. The sites are further discussed Hamerow, H. (2012).

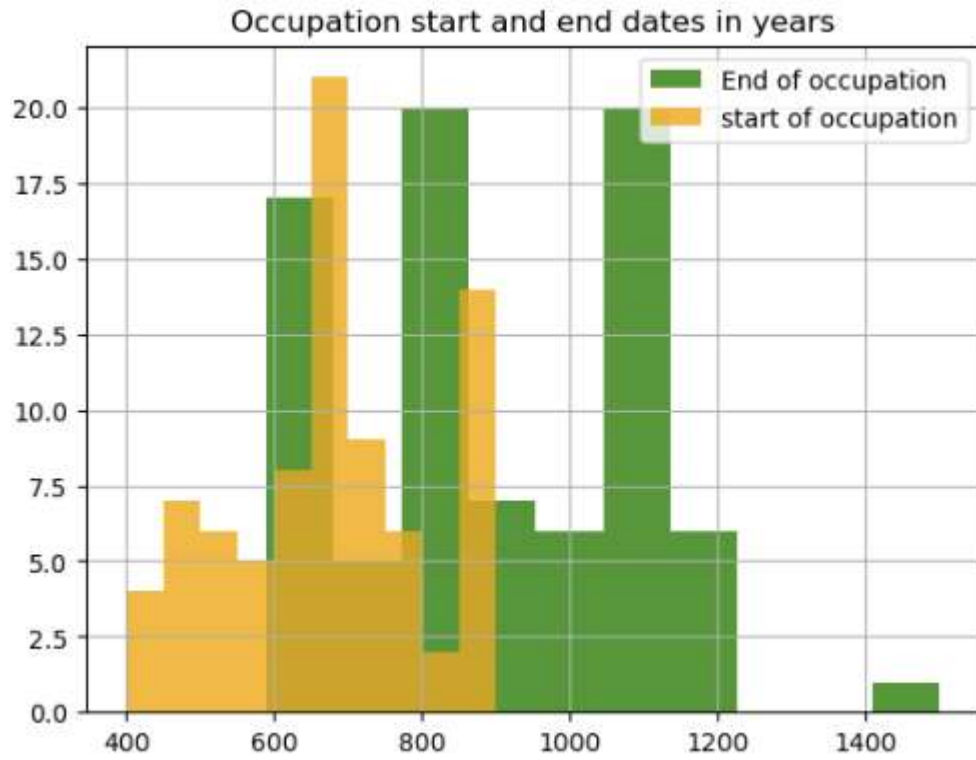


Figure 3-19 Settlement Occupation Dates

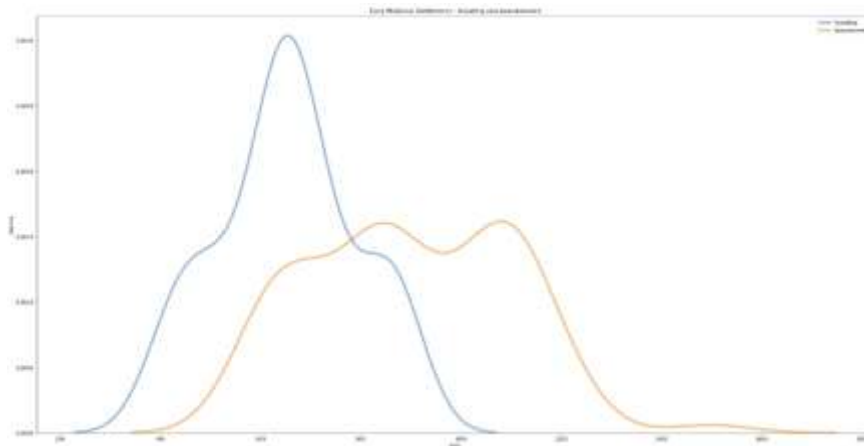


Figure 3-20 Kdeplot of Settlement Occupation Dates

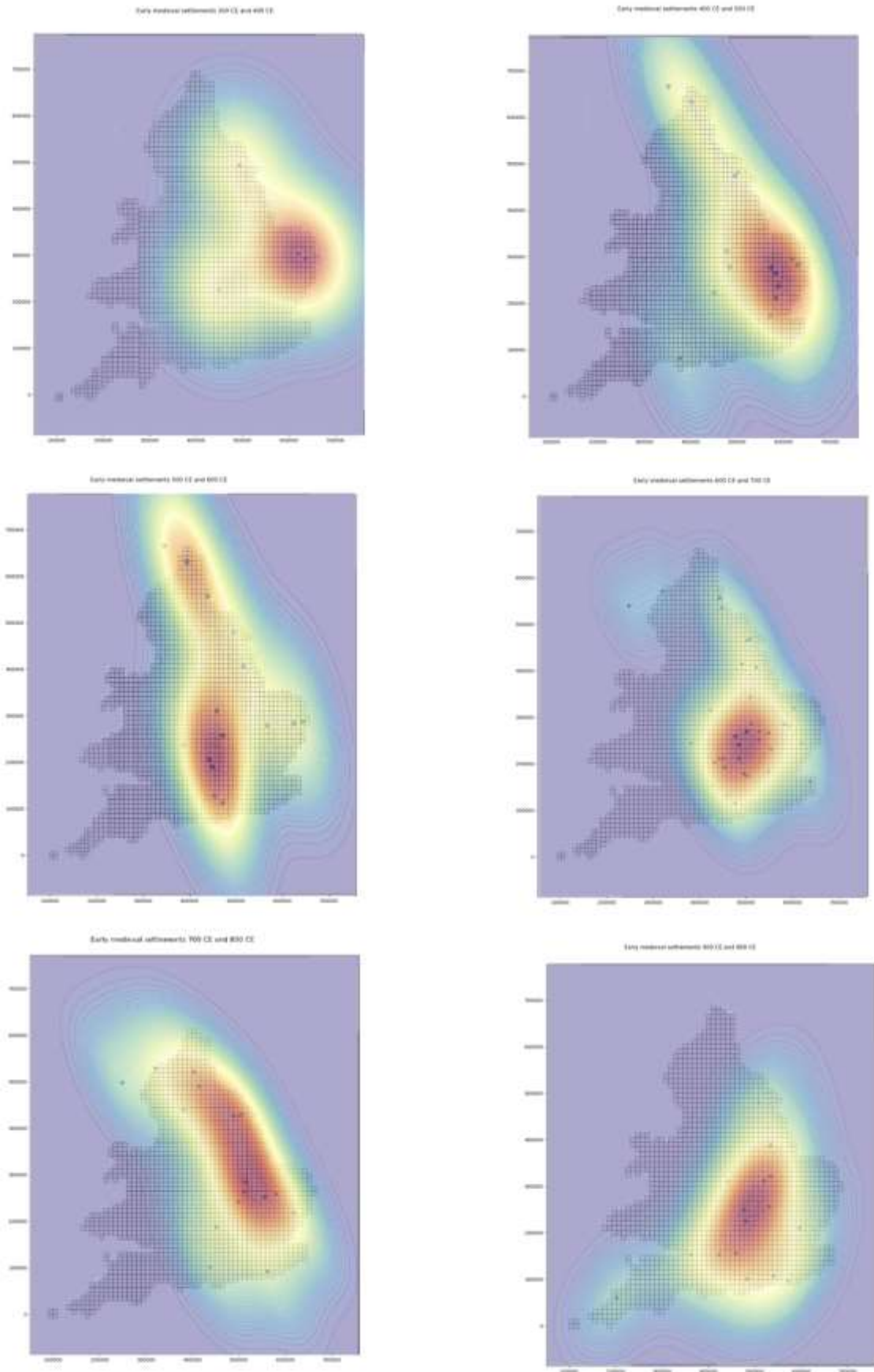


Figure 3-21 Spatial Variation of Occupation Date

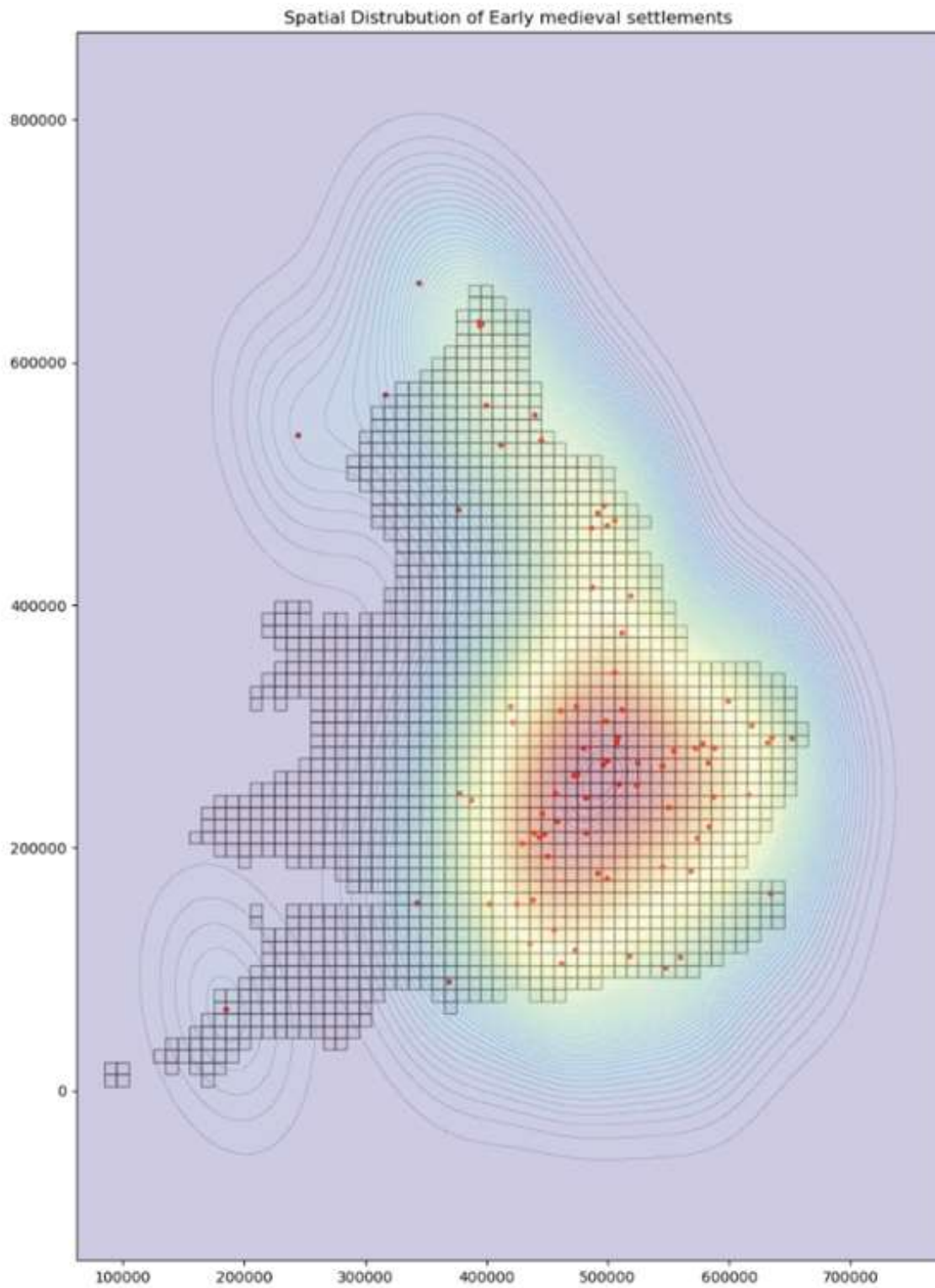


Figure 3-22 Spatial Distribution of Early Medieval Settlements

Bioarchaeological Data

Archaeological evidence supports the productivity of Anglo-Saxon agriculture through various types of data, including bioarchaeological, palaeoecological, and geoarchaeological evidence. For example, the FeedSax project has provided a comprehensive dataset that shows an increase in arable farming between the 8th and 13th centuries in England. This highlights the significant role of cereal cultivation during this time (McKerracher et al., 2023). The shift towards open-field farming during the early Middle Ages further confirms the expansion, which facilitated the so-called Medieval Agricultural Revolution (Hamerow et al, 2019). Specific studies, such as those conducted at Lyminge, Kent, reveal the presence of palaeochannels and organic deposits indicating continuous landscape management and cereal cultivation during the late Romano-British period and the Anglo-Saxon era, suggesting a stable and productive agricultural system (Maslin, 2019).

Additionally, charred plant remains from the Mid Saxon period (c. AD 650–850) challenge the 'bread wheat thesis' and show the increased importance of rye and oats, indicating a diversification in crop production and regional variations in agricultural practices (McKerracher, 2016). The discovery of a productive site in south Oxfordshire, marked by coins and other finds, suggests the existence of a trading point that likely facilitated agricultural exchange and surplus production during the 8th to 9th centuries (Mileson and Brookes, 2014).

The transformation in agricultural practices during the Mid Saxon period is evident from settlement, zooarchaeological, and archaeobotanical evidence, including the increased cultivation of bread wheat and specialized sheep's wool production (McKerracher, 2014). Environmental archaeological evidence from Flixborough provides insights into the local and regional agricultural economy, revealing shifts in resource exploitation and trade networks,

which underline the economic and social changes during the 9th century (Dobney et al, 2007). Furthermore, the continuity in land management and vegetation history from the late Romano-British to medieval periods at Lyminge demonstrates the long-term stability and productivity of Anglo-Saxon agricultural systems.

Finally, the broader context of agricultural productivity is highlighted by the significant increase in output during the so-called 'agricultural revolution,' which allowed English farmers to feed a growing population without massive imports, indicating a substantial rise in productivity. Together, these diverse lines of evidence present a comprehensive picture of Anglo-Saxon agricultural productivity, characterized by innovations in crop and livestock management, regional adaptations, and the integration of agricultural practices into broader economic and social frameworks. Collectively, these diverse lines of evidence paint a comprehensive picture of Anglo-Saxon agricultural productivity, marked by innovations in crop and livestock management, regional adaptations, and the integration of agricultural practices into broader economic and social frameworks.

Changes in land use patterns, such as the expansion of cultivated land or the construction of irrigation systems, can indicate population growth and increased demand for food. Crop yield estimates based on historical records or paleoclimatic data can help calculate the potential food supply and the number of people it could support. The analysis of animal bones found in archaeological sites can provide information about livestock populations and the availability of meat resources, which can, in turn, be related to human population size. The analysis of pollen grains preserved in sediments can reveal changes in vegetation patterns over time. Deforestation or the expansion of agriculture can indicate population growth and increased land use.

The Bioarchaeological database for the FeedSax project (McKerracher et al., 2023) is the digital archive for Feeding Anglo-Saxon England (FeedSax): The Bioarchaeology of an Agricultural Revolution Project. It provides extensive agricultural data and uncovered significant insights into the agricultural and dietary practices of the Anglo-Saxon period, particularly between the 8th and 13th centuries.

The project revealed that the expansion of arable farming during this period was closely linked to the spread of open-field farming systems, which facilitated a major increase in cereal cultivation across large parts of Europe, including England (Hamerow et al, 2019). This expansion enabled landowners to amass wealth by exploiting low-input cereal farming regimes and expanding the amount of land under cultivation, particularly during the mid-Saxon period.

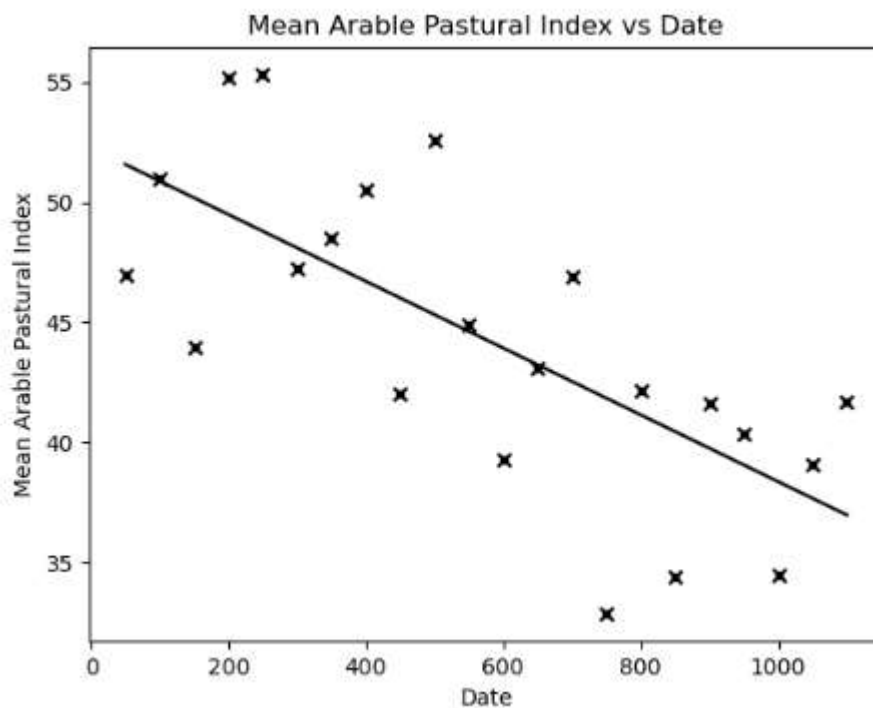


Figure 3-23 Arable - Pastoral Index vs date

The project also identified a shift to low-input cultivation regimes and the probable expansion onto heavier soils in the Late Saxon period. Evidence suggests that crop rotation was practised from at least the mid-tenth century (Hammerow et al., 2020).

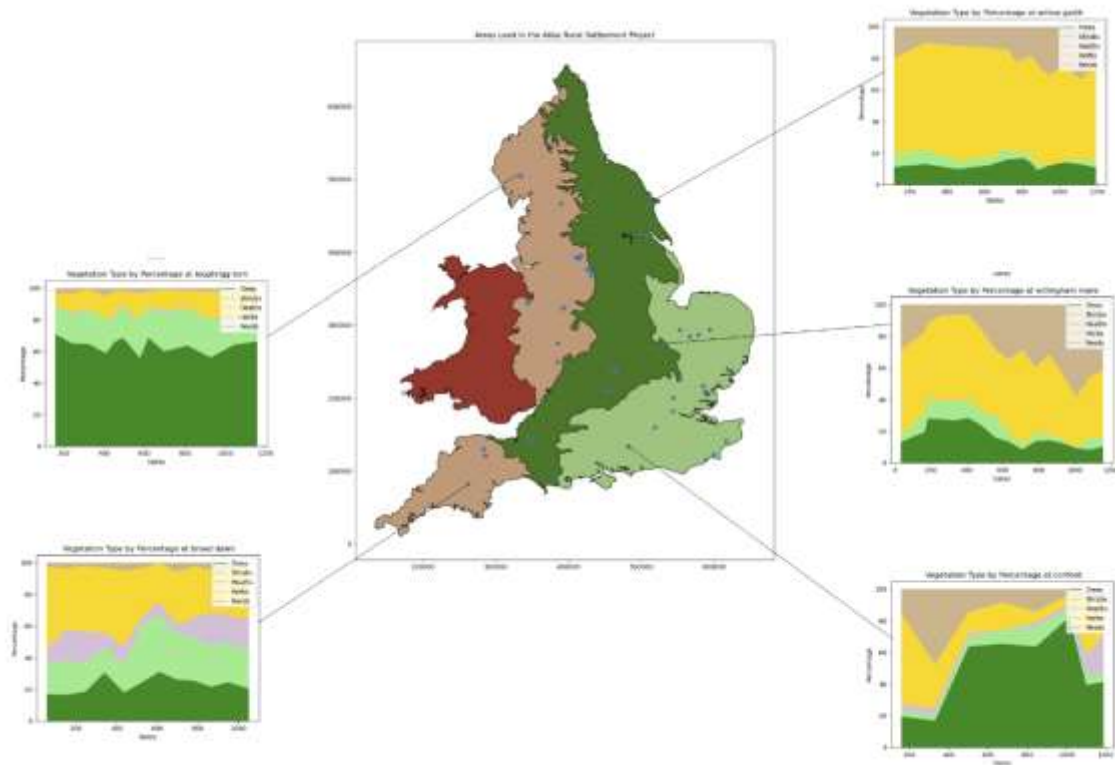


Figure 3-24 Change in Vegetation at Selected FeedSax Sites

Additionally, stable isotope analysis from various sites, such as Broughton Lodge and Raunds Furnells, provided insights into the diet and weaning practices of the population. It indicated a

diet primarily composed of terrestrial animal protein with some freshwater fish and a weaning age around two years old.

The analysis of lipid residues from pottery in Hamwic showed that the population consumed ruminant fats, leafy vegetables, and minor amounts of aquatic foods, reflecting a diverse diet. Furthermore, the project highlighted the role of food in social identity and cultural practices, with evidence suggesting that while elaborate cuisine may not have been a distinct concept, there were specialized food preparations and privileged foods for social elites. The study also noted the impact of Christianity on dietary practices, such as the reduction in consumption of horse meat, which was influenced by religious beliefs and varied across different regions and social identities. Overall, the FeedSax project provides a comprehensive understanding of the agricultural, dietary, and social dynamics of Anglo-Saxon England (McKerracher, 2018).

The works by Van Der Veen (2013; 2022) provide data about the agricultural transition from Roman Britain through the Early Medieval period and into the Late Medieval period, which is very valuable for understanding the agrarian economics of the Anglo-Saxon period.

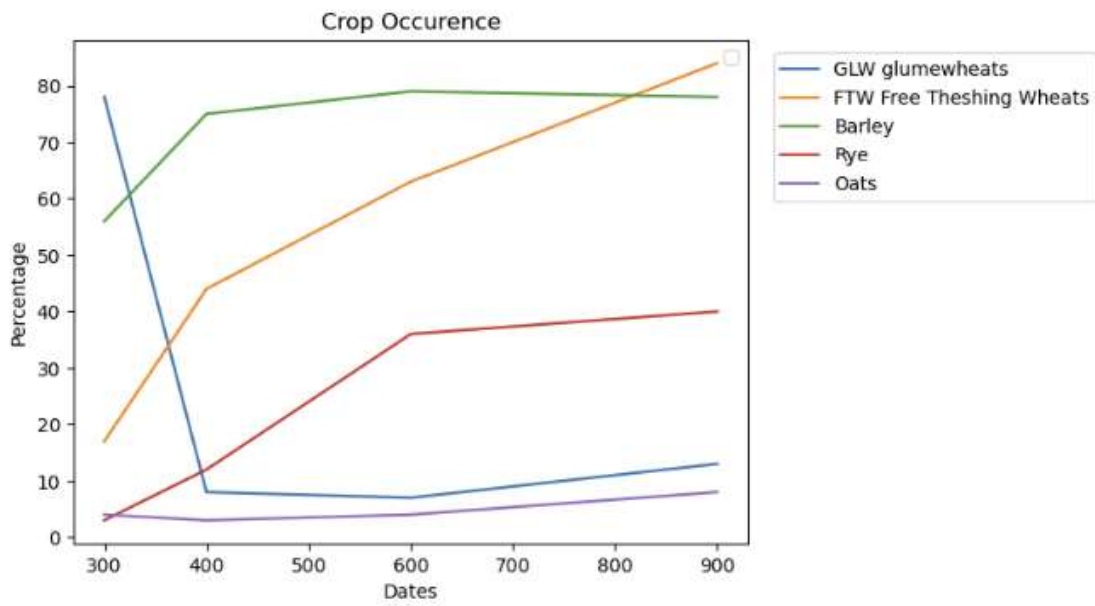


Figure 3-25 Occurrence of Crops

It has been shown (Galofré-Vilà, 2017) that average height is a good proxy for agricultural production.

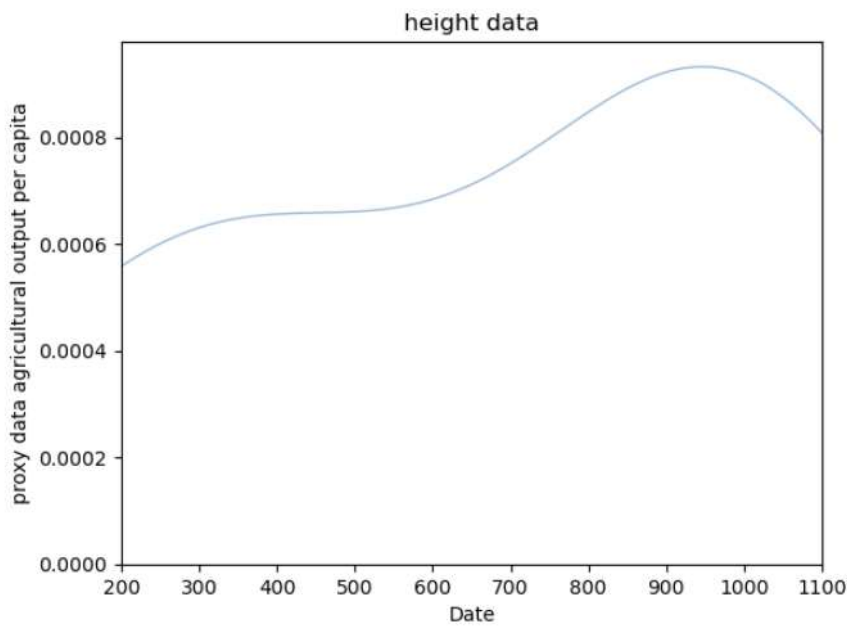


Figure 3-26 proxy data for per capita agricultural output vs Date

Cemetery and grave data

The size and number of burials in cemeteries can provide a rough estimate of population size, although factors like burial practices and preservation need to be considered. Whilst demographic analysis of the age and sex distribution of skeletal remains can offer insights into birth and death rates, which can be used to estimate population growth or decline. The quantity and quality of goods buried with individuals can indicate their social status and wealth disparities within a community (Brownlee, 2021).

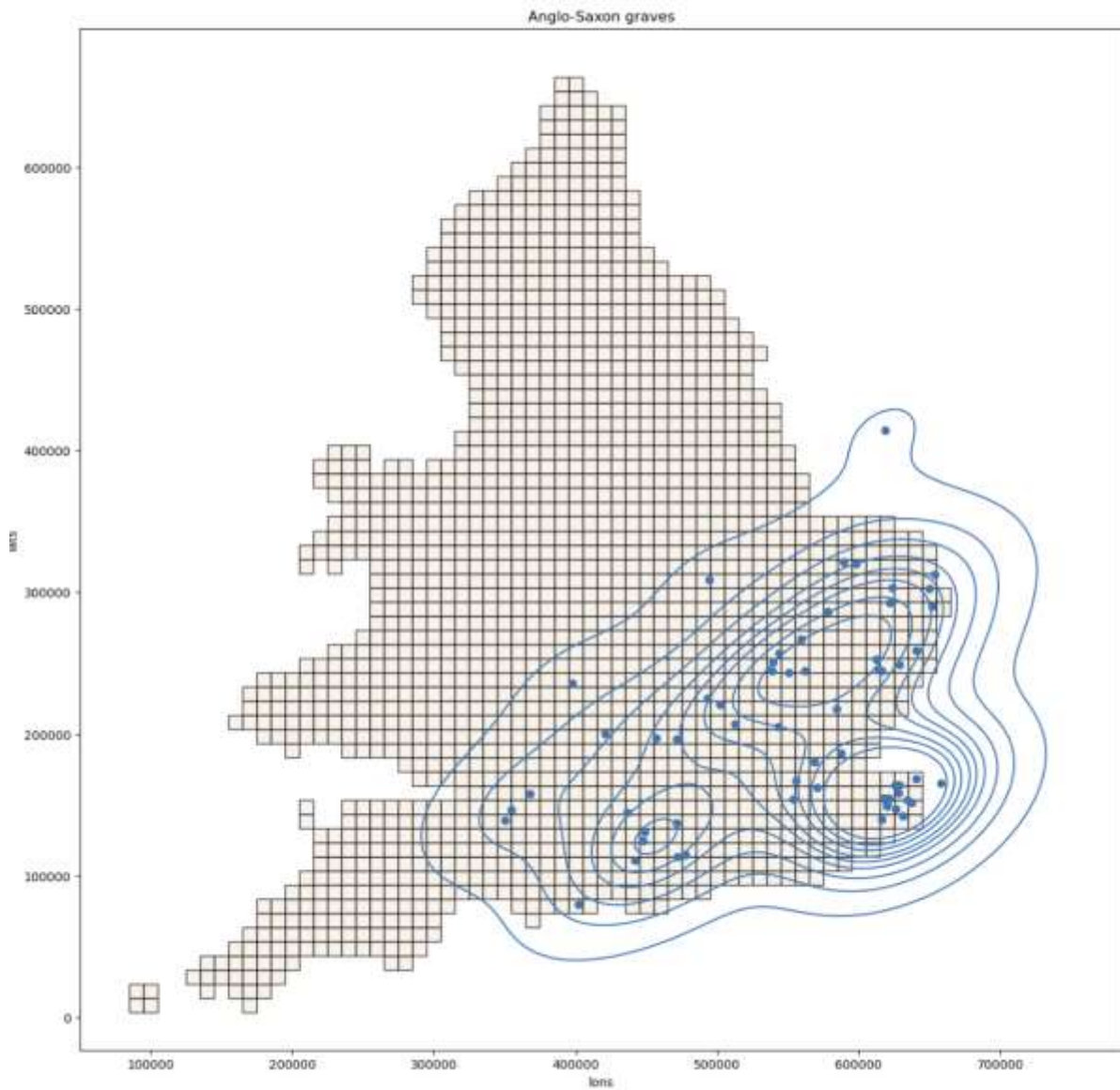


Figure 3-27 Anglo-Saxon graves Burial Practices (Brownlee, 2021)

The Anglo-Saxon Kent Electronic Database (ASKED) project collates the archaeological evidence for the Anglo-Saxon populations of east and west Kent AD 400-750 and consists of the human skeletal remains, the grave goods and the burial structures from inhumation cemeteries (Harrington and Brookes, 2012).

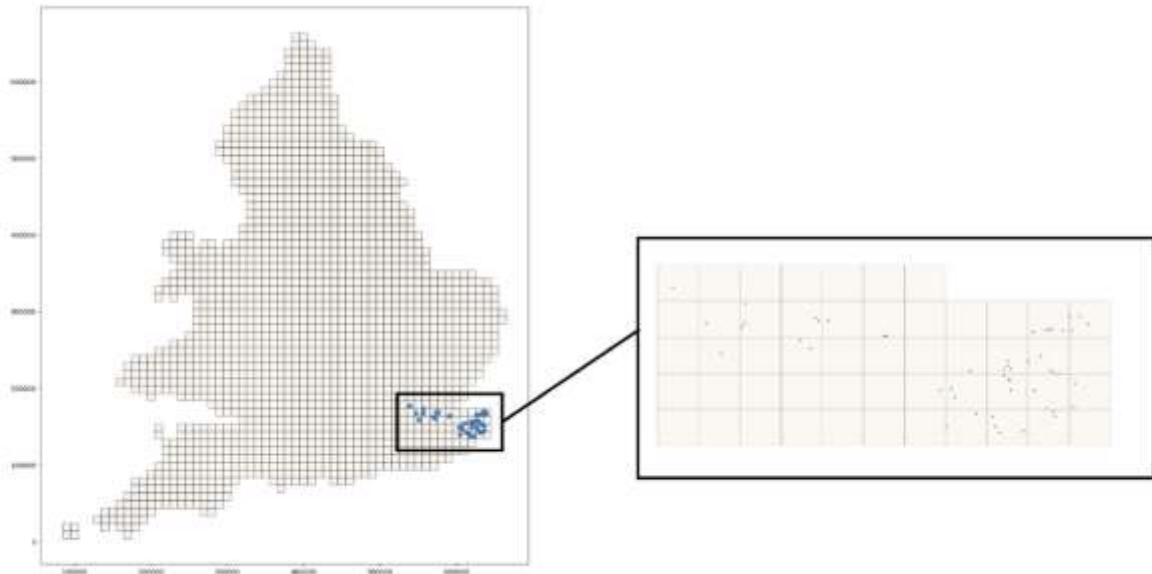


Figure 3-28 Spatial Distribution of the ASKED Project

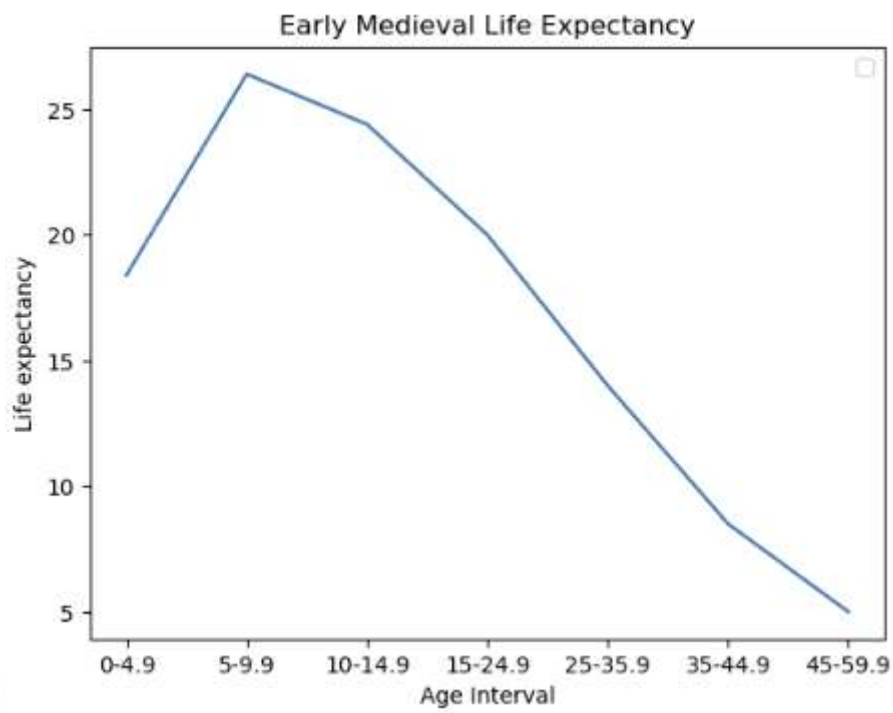


Figure 3-29 Early Medieval Life Expectancy

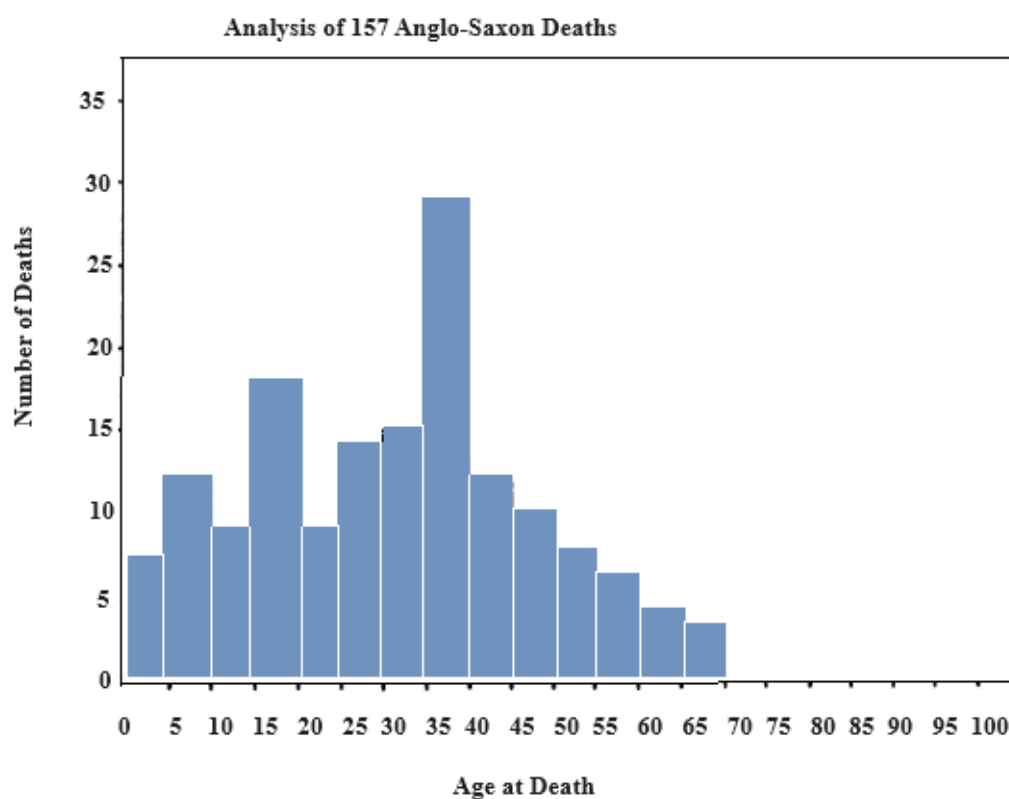


Figure 3-30 Distribution of Age at Death

Textual analysis

The Anglo-Saxon Chronicles

Human activities' intensity and location change over time due to demographic and cultural factors such as population levels, farming practices, and other economic activities. Understanding these changes over the long term provides insights into the cultural evolution of historical societies. Frequency analysis of collected evidence, such as historical texts and archaeological materials, provides a reasonable proxy for the past activity that produced these artefacts (McLaughlin, 2018).

This research analyses the frequency of entries in the Anglo-Saxon Chronicles and other texts. Archaeological evidence from the early medieval period in England can provide valuable information and enable data triangulation for the patterns revealed by textual analysis. Spatial and temporal statistical methods, such as cluster analysis, can help discover long-term trends and are increasingly used by archaeologists. Combining these analyses with historical data containing contextual and chronological information provides a richer and more rounded picture of the early medieval period in England.

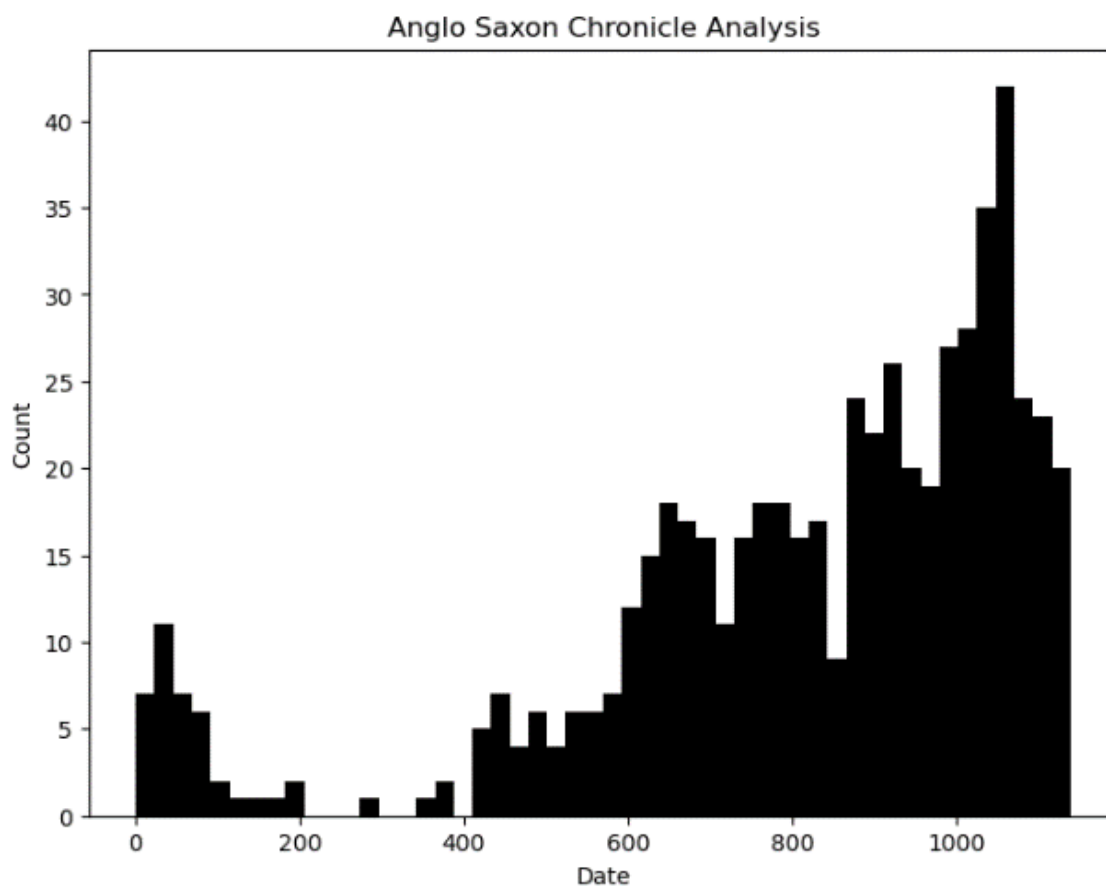


Figure 3-31 Word count vs time for the Anglo-Saxon Chronicles

It is also possible to assess the changing influence of the Anglo-Saxon kingdoms over time.

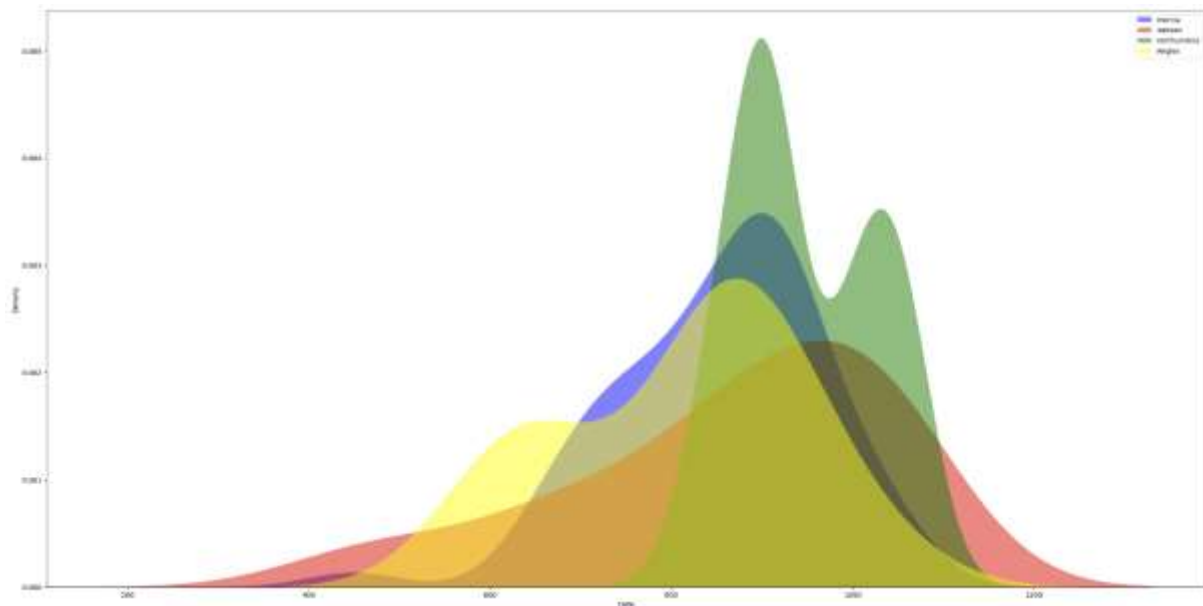


Figure 3-32 Comparing Anglo-Saxon kingdoms influences over time.

Early Medieval Battles

The Battlefield Trust has a database of all the documented battles in the early medieval period, including dates and locations.

The Anglo-Saxon period in England, which lasted from the fifth to the late eleventh century, witnessed many significant battles that played a crucial role in shaping English history. One of the earliest recorded battles is the Battle of Badon Hill, estimated to have taken place around 500 CE, where the Britons, possibly led by King Arthur, successfully defended against the advancing Saxons.

In 577 CE, the Battle of Deorham saw the West Saxons, led by Ceawlin, defeat the Britons, leading to the capture of important cities such as Gloucester, Cirencester, and Bath, significantly expanding Saxon territory. The Battle of Edington in 878 CE was another pivotal conflict, where Alfred the Great of Wessex defeated the Viking Great Heathen Army, resulting in the Treaty of Wedmore and the establishment of the Danelaw in England.

The Battle of Maldon in 991 CE, immortalized in an Old English poem, witnessed the Anglo-Saxons under Byrhtnoth facing a Viking invasion, ultimately resulting in a Viking victory and substantial tribute payments by the English. The Battle of Assandun in 1016 CE played a crucial role in the struggle between Edmund Ironside and Cnut the Great, culminating in a decisive victory for Cnut and the eventual unification of England under his rule.

The Battle of Stamford Bridge in 1066 CE marked the end of the Viking Age, as King Harold Godwinson of England defeated the Norwegian King Harald Hardrada, only to face the Norman invasion shortly after. The Battle of Hastings, also in 1066 CE, was a turning point where William the Conqueror defeated Harold Godwinson, leading to the Norman Conquest of England and the end of the Anglo-Saxon era.

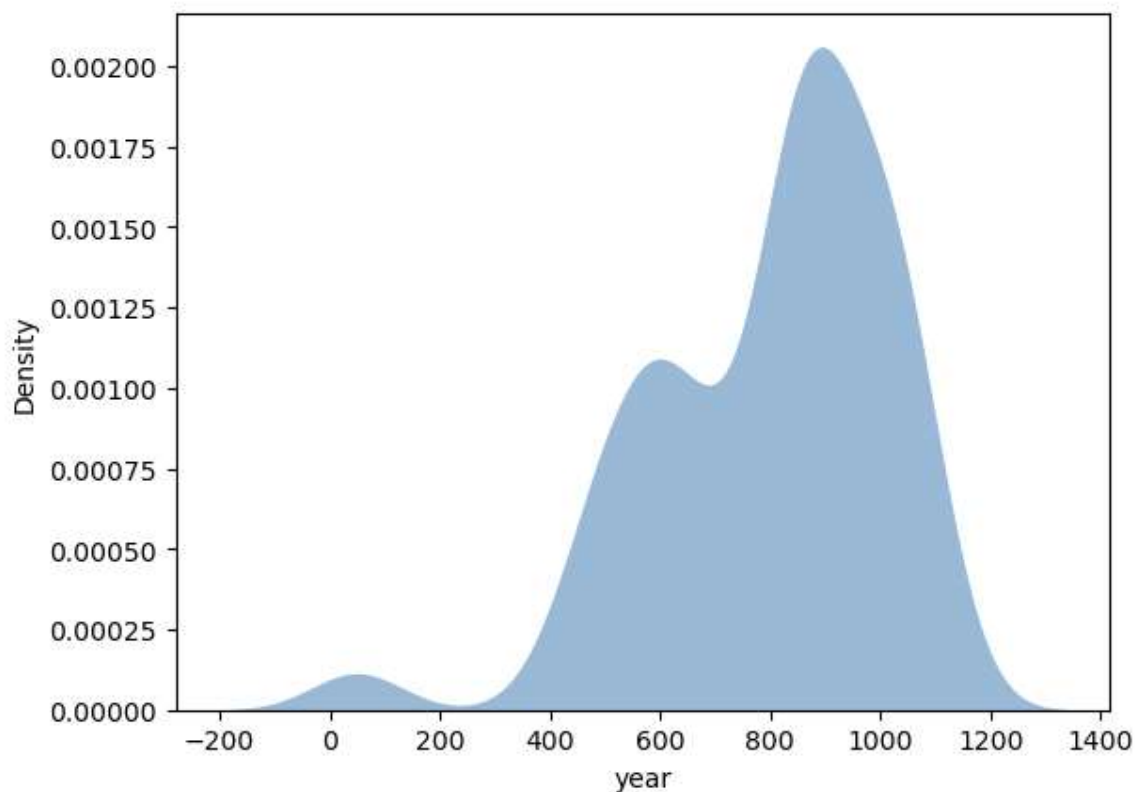


Figure 3-33 Battles from the Anglo-Saxon Chronicles

These battles, recorded in sources such as the Anglo-Saxon Chronicle and other historical texts, highlight the turbulent and transformative nature of the Anglo-Saxon period. Each conflict significantly contributed to shaping the political and cultural landscape of medieval England. Furthermore, the lasting impact of these battles on the development of the English state is evident in the establishment of fortified burhs and the evolution of military strategies and governance structures. The cultural memory of these battles, preserved in literature and historical records, continues to influence the understanding of Anglo-Saxon history and identity.

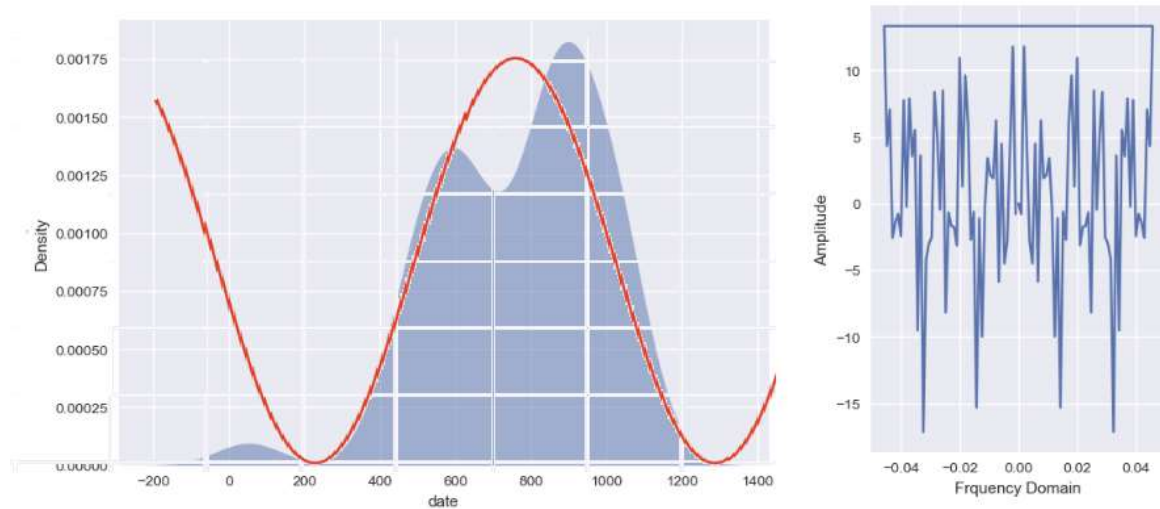


Figure 3-34 FFT of Anglo-Saxon Battles

Chapter 4 Content and Results

Proxy variables

Population

Negligible direct data exist concerning the population and distribution of early medieval England, so proxy data and demographic modelling are necessary. The post-Roman, early medieval and post-Norman conquest population distributions can be approximated by the distribution of archaeological finds from the period (Allen, 2015; Green, 2017)

The population over the early medieval period varied from a maximum of approximately 3.5 – 4 million during 350 – 400 CE decreasing to roughly 0.5 million during the period 450 – 550 CE and subsequently rising to around 1.7 million during the period 600 – 1100 CE when the Domesday book was compiled. The population during the period 600 – 1100 CE is not known with any degree of certainty, although archaeological and historical evidence and demographic modelling can give indications, although these are sensitive to the modelling assumptions and interpretations of the evidence (Härke, 2002).

An analysis of the archaeological and historical data showed that the intensity of archaeological artefacts and historical events varied temporally and spatially. The population during the early medieval period, as measured by the proxy data such as radiocarbon sites and PAS early medieval sites, shows a clear temporal variation with an approximate period of 300 years as determined by Fast Fourier Analysis (FFT) of the data, as shown below:

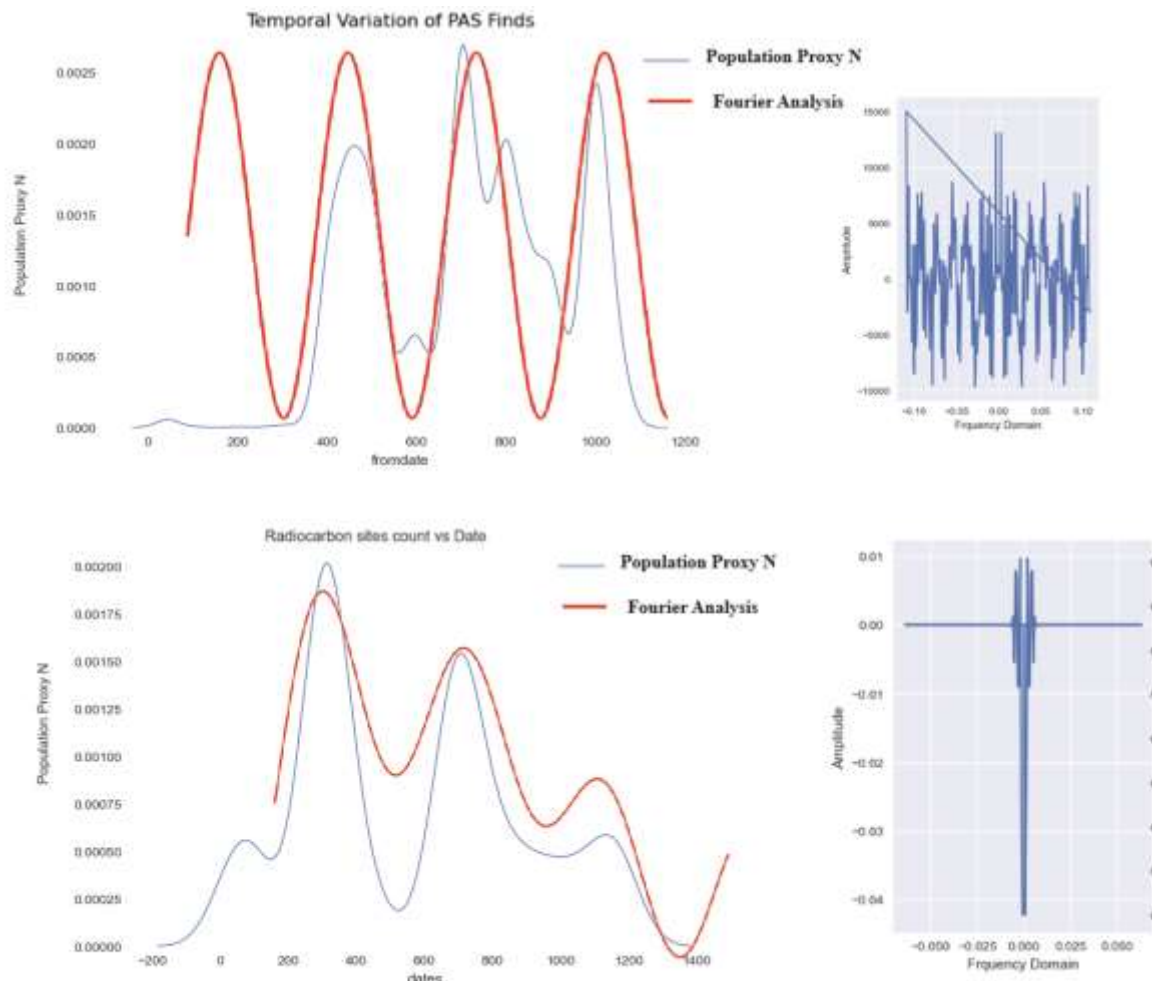


Figure 4-1 Fourier analysis of population proxy data

Warfare Intensity

The Warfare intensity W is measured using the battle dates given by the Anglo-Saxon chronicles and the temporal variation of Early medieval non-coin hoards which have approximately the same variation.

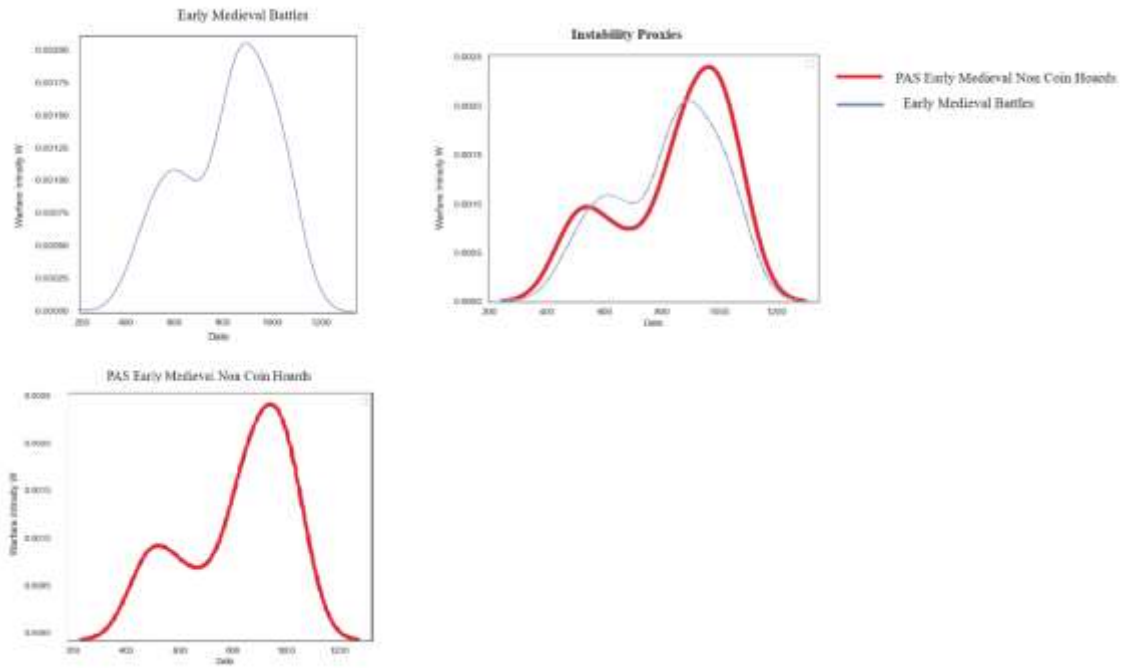


Figure 4-2 Warfare intensity proxy data

State Strength

The State Strength S is measured by using PAS Early Medieval Coin Hoards and EMC Early Medieval Single coin finds.

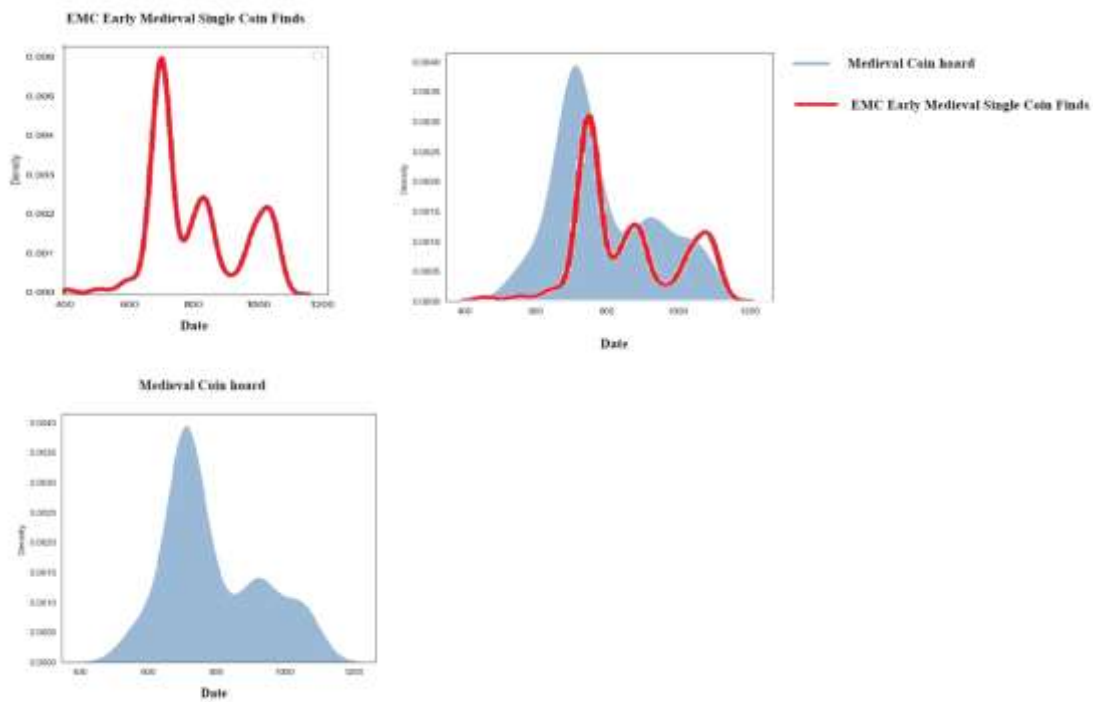


Figure 4-3 State Strength proxy data

Mathematical Analysis

The mathematical analysis of oscillating systems of this type was developed to study the population dynamics of predator-prey systems. The logistic equation describes the growth of a population determined by the carrying capacity K of the local territory.

$$dN/dt = rN(1 - N/K) \quad 4.1$$

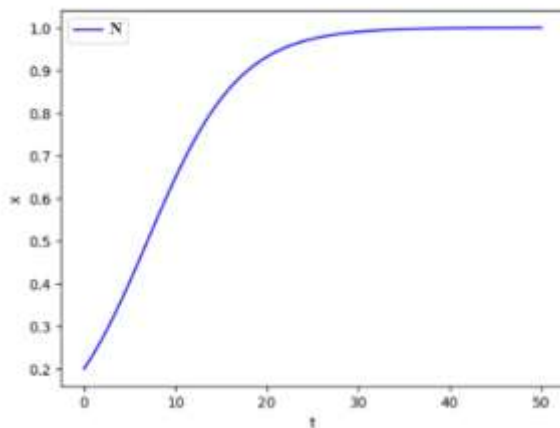
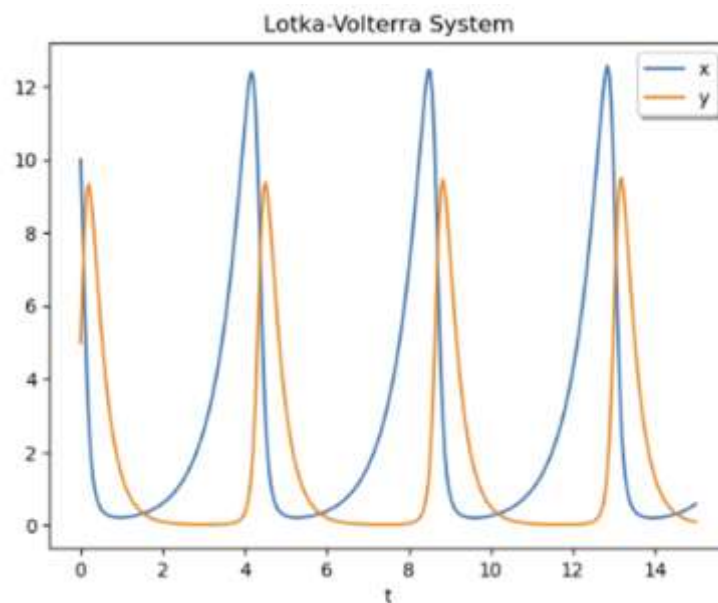


Figure 4-4 Logistic Equation

The Lotka-Volterra equations exist in various forms. They can be used to model the complex dynamics of medieval agrarian states by describing them as oscillating dynamic systems



parameterised by geopolitical, economic, and military variables.

Figure 4-5 Lotka-Volterra Predator-Prey System with Oscillations

The predator-prey equations are a type of Lotka-Volterra equations used to model renewable resources and their consumption and is frequently used to model ecosystems.

$$dx/dt = ax - bxy \quad 4.2$$

and

$$dy/dt = cxy - dy \quad 4.3$$

These equations can be modified so that more than two species can interact with each other, giving the Lotka-Volterra competition model.

The Lotka-Volterra competition equations can be applied directly to the interactions of the early medieval kingdoms. An analysis of the expansion of Wessex Kingston using data from early medieval single coin finds shows that these equations model the form of the interaction well.

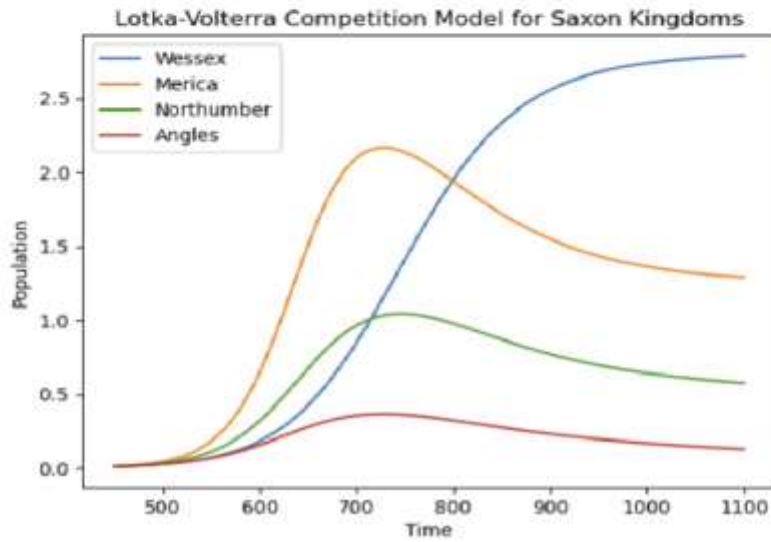
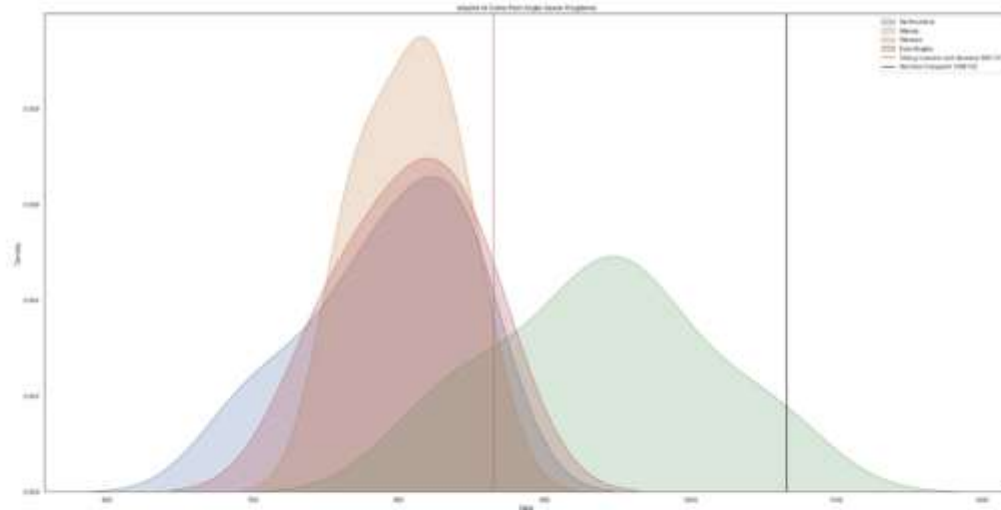


Figure 4-6 Modelling the Competition of Anglo-Saxon Kingdoms using the Lotka-Volterra Competition Equations.

For Lotka-Volterra predator-prey type systems Turchin (2006, p.153) suggests that there are oscillations of approximate period:

$$T \sim 2\pi/\sqrt{r\delta}$$

4.4

Which for a population growth rate of ~ 0.02 gives a period of approximately 300 years. Applying this to the Malthusian dynamics of pre-industrial agrarian society, Nefedov (2013) suggests that the dynamics of the agricultural population have an oscillating nature. Theoretically, these oscillations are heavily damped, and the system would tend to an equilibrium state. However, random factors such as crop yield disturb the system equilibrium and prevent this from occurring.

The population dynamics are given by the logistic equation:

$$dN/dt = rN(1 - N/K) \quad 4.5$$

Where K is the carrying capacity - the maximum size of a population that may live in this territory.

The stock growth is given by:

$$dK/dt = aN/(N + d) - N \quad 4.6$$

Where \mathbf{a} and \mathbf{d} are constants. Solving these coupled equations gives a damped oscillating system with period:

$$T = 2\pi / \sqrt{(r - r/q - \frac{r^2}{4})} \quad 4.7$$

Where $\mathbf{q} = \mathbf{a/d}$,

The period T decreases when r and q increase and increases accordingly when these values decrease.

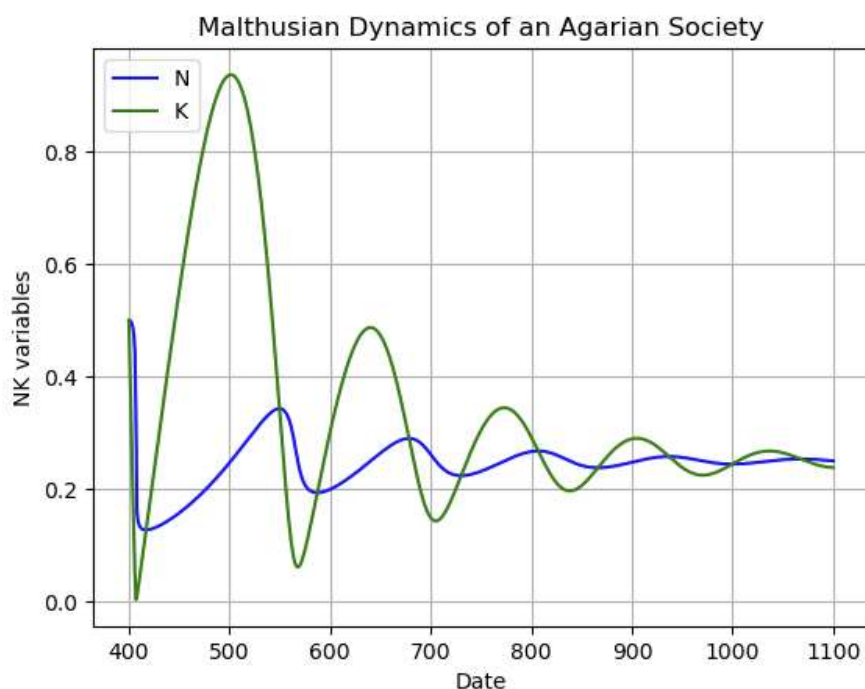


Figure 4-7 Malthusian dynamics of an agrarian society

This model when applied to the study of early medieval England and the Anglo-Saxon period, suggests that a population's nutritional status should vary inversely with population size based on the Malthusian relation between population and living standards.

Demographic Structural Theory

Secular Cycles

The theoretical background describing the relationship between phases of secular cycles is based on Turchin's demographic-fiscal model (Turchin, 2003), which was based on Goldstone's work (Goldstone, 1991). This model can generate a significant number of relevant theoretical predictions. The main logic of this model is that during the initial phase of the demographic cycle, per capita production and consumption levels are notably elevated. Consequently, there is a surge in both population growth rates and surplus production. This stage is characterized

by a population that can fulfil tax obligations with ease, making tax collection a straightforward process, so the growth of government revenues is directly linked to population expansion.

Plotting the Population N and the Warfare intensity W gives:

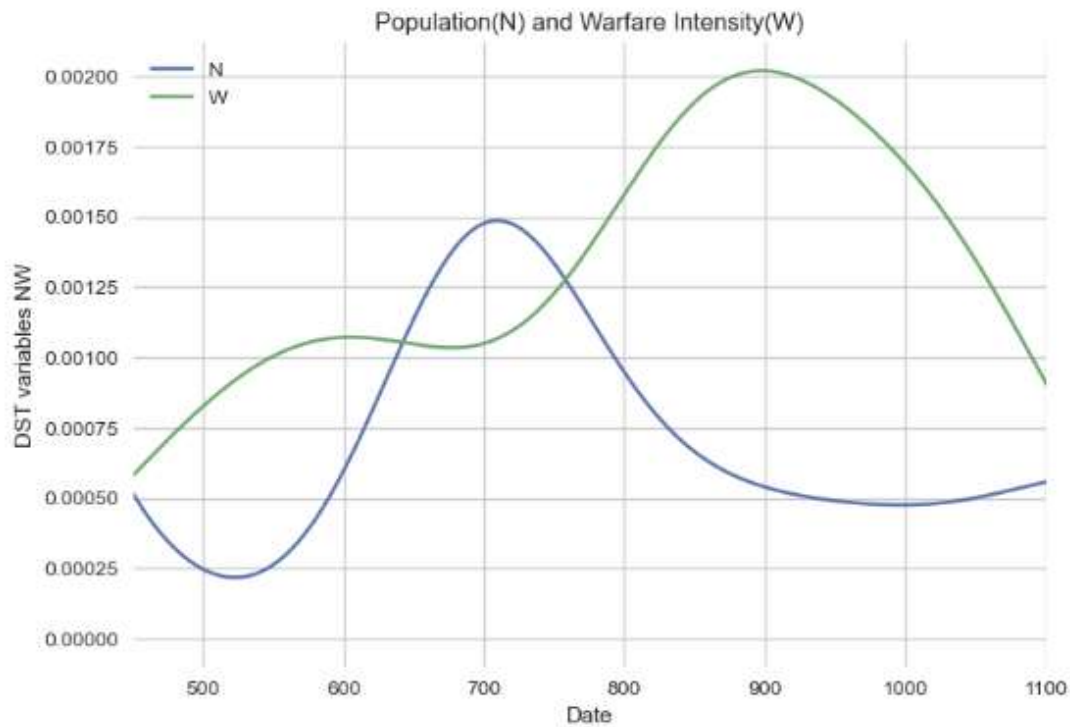


Figure 4-8 Plot of measured NW data

During the intermediate phase, as the population expands, production and consumption per capita tend to decrease. Tax collection becomes increasingly difficult, so state revenue levels out. At the same time, government expenditures continue to rise because the state needs to support an increasing population. As a result, during this phase, the government may face various economic problems.

Plotting the Population N and the State Strength S gives:

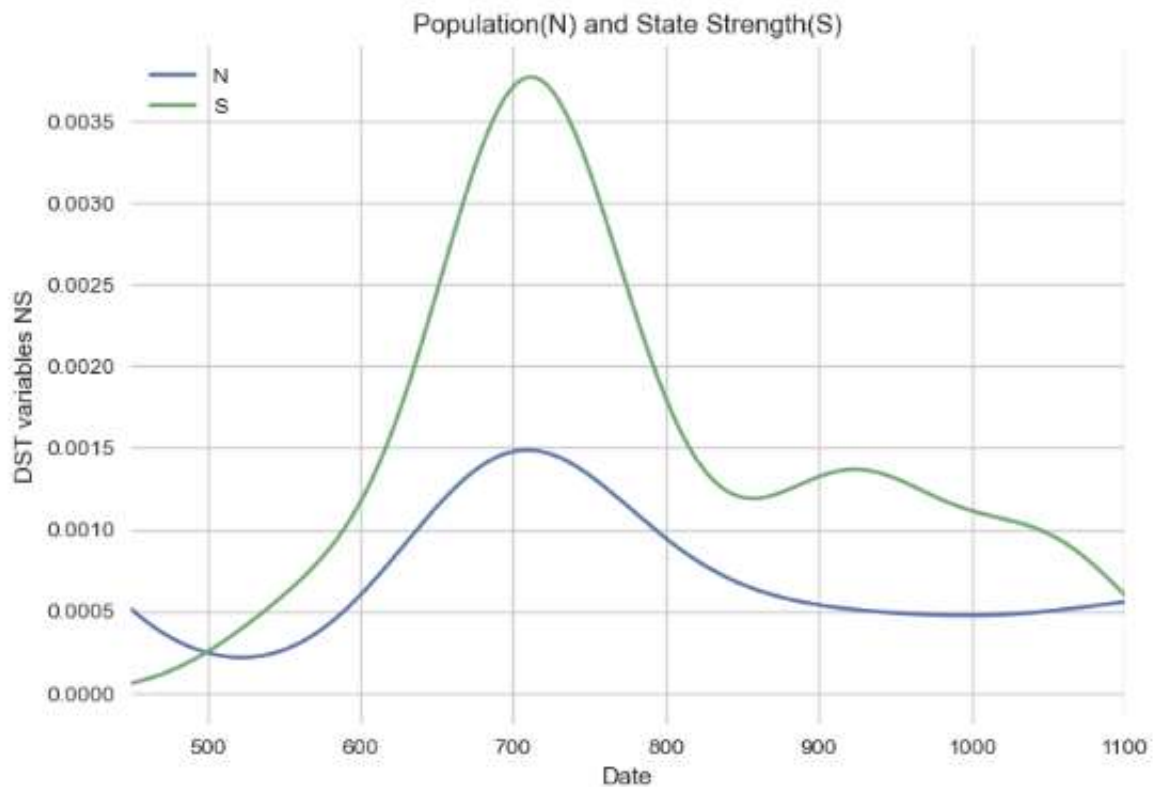


Figure 4-9 Plot of measured NS data

As the population grows, the production of goods per person decreases, following the ‘law of diminishing returns’, leading to a decline in the surplus produced and a reduction in the government's income. Meanwhile, the government requires more resources to manage the decelerating population growth. Eventually, this scenario culminates in the government's collapse and a demographic crisis, followed by a new cycle of population fluctuations (Korotayev, 2016).

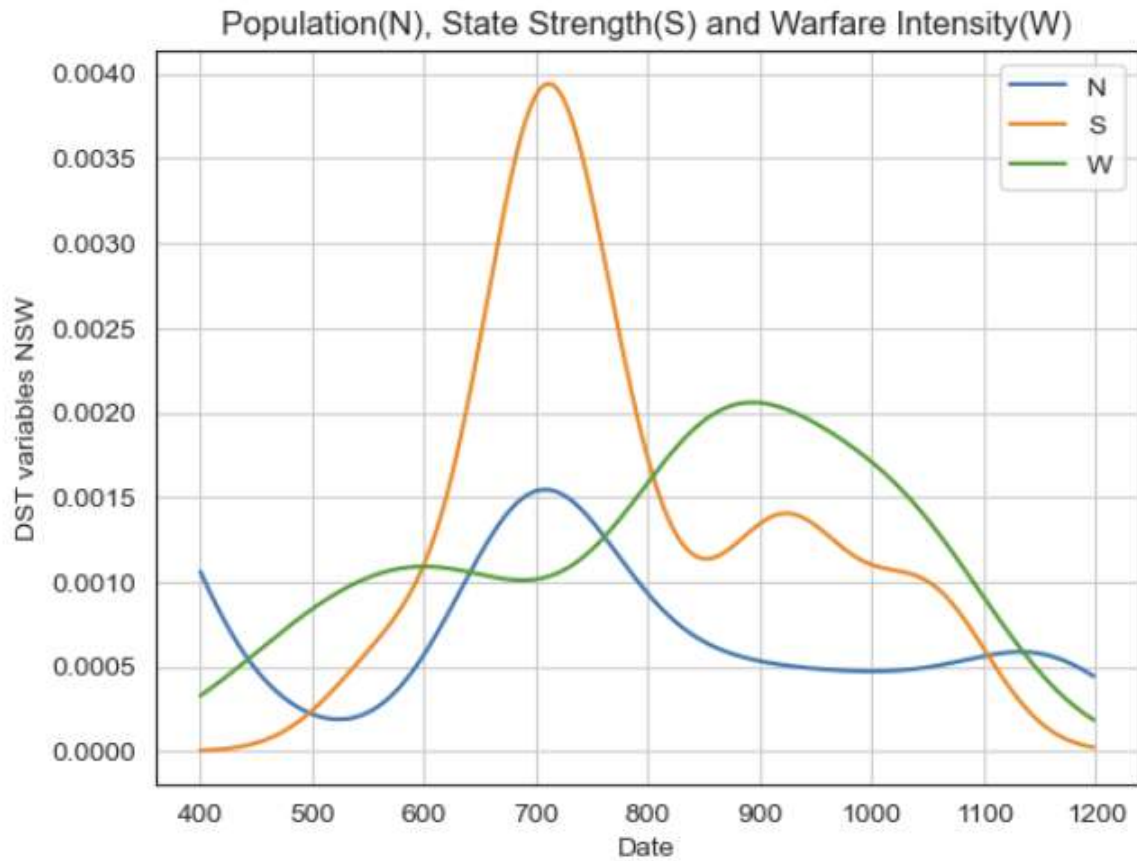


Figure 4-10 Plot of measured NSW data

Demographic Fiscal (DF) Model: Population and State Strength

The simplest of the mathematical models for secular cycles is the demographic-fiscal (DF) model (Turchin, 2003). This model has two state variables: N, population density, and S, state strength and is described by the following equations:

$$dN/dt = N(1 - N/k) \tag{4.8}$$

and

$$dS/dt = N(1 - N/k) - \beta N \tag{4.9}$$

where,

$$k = c S / (s_0 + S)$$

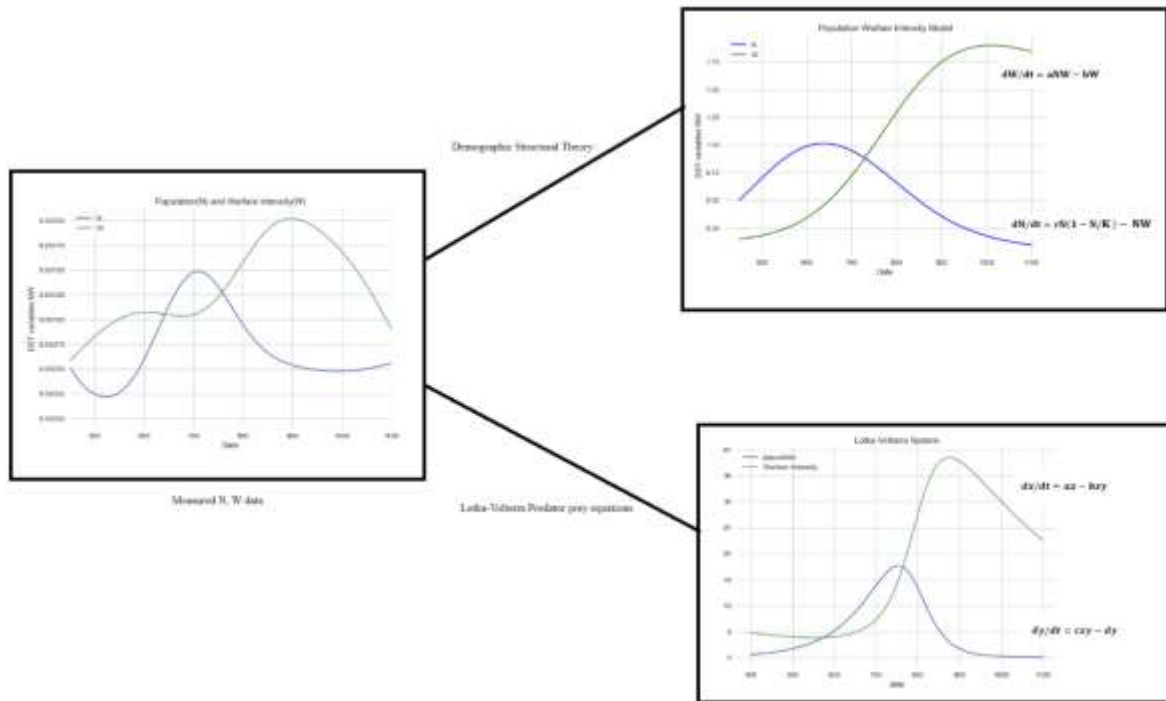


Figure 4-11 Cliodynamics Theory for NS

As seen in Fig. 4-12 below, the early Anglo-Saxon period was dominated by The Anglo-Saxon kingdom of Mercia for approximately 300 years, between 550 CE and 879 CE. During this period, Mercia annexed or gained submissions from five of the six kingdoms of the Heptarchy: East Anglia, Essex, Kent, Sussex, and Wessex and effectively controlled England south of the Humber estuary. At the end of the 9th century, following the invasions of the Vikings and their Great Heathen Army in 865 CE, Danelaw absorbed much of the former Mercian territory. The final Mercian king, Ceolwulf II, died in 879 CE, with the kingdom losing its political independence. This was the first secular cycle within the Anglo-Saxon period.

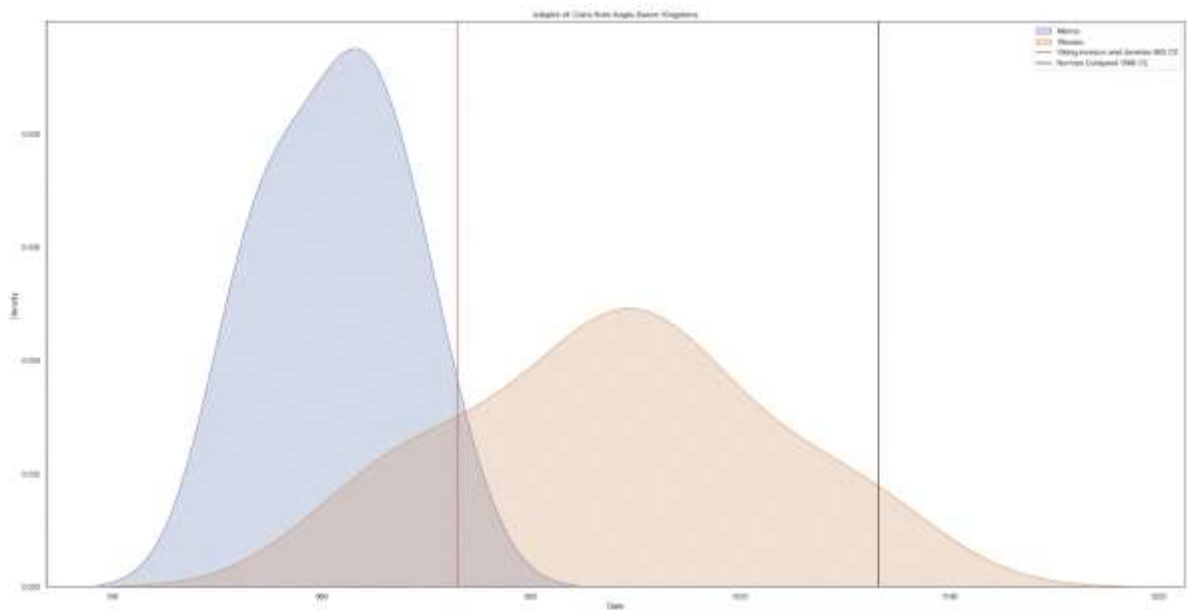


Figure 4-12 The Mercia and Wessex cycles

In the work by Alexander (2016), a Wessex-dominated Anglo-Saxon cycle is stated to have started at the end of the ninth century and continued into the late eleventh century, corresponding to the beginning of the high medieval cycle. The cycle is proposed to have begun with Alfred the Great's victory at Edington in 878, which re-established Anglo-Saxon control over Wessex after the Danish invasion in 865 had overrun East Anglia, Northumbria, and half of Mercia. Over the next seventy years, three generations of Alfred's family conquered all of England, indicating an expansion phase with a rising population, pacified elites, and increasing state strength. The period from 880 CE to 950 CE was the expansion phase of the Wessex-dominated Anglo-Saxon cycle.

The Anglo-Saxon period ended with the Norman Conquest. The Danes had previously conquered England, signalling a period of disintegration. However, the crisis of 957 CE was resolved without bloodshed, indicating a trend towards integration at that time. There are no

records of internal instability events between 975 CE and 1015 CE, which suggests that the disintegrative trend may not have begun much earlier than 1013 CE.

The political history of England in the 11th century indicates heightened competition and conflict among the elite. The Danish conquest in 1013 occurred in less than a year, showing significant political divisions among the elites. A power struggle between King Edward the Confessor and Earl Godwin almost led to a civil war in 1051 CE. When Edward the Confessor died in 1066 CE, he was succeeded by Earl Harold Godwinson, whose claim to the throne was challenged by Harald Hardrada of Norway and William of Normandy. These events suggest that 11th-century Anglo-Saxon England suffered from a degradation of state legitimacy, making royal succession difficult and leaving the country vulnerable to conquest (Alexander, 2016).

Population and warfare intensity

The model has two state variables: N , population density, and W , warfare intensity (Turchin and Korotayev, 2006). The equation for N assumes that in the absence of war, the population will grow logistically and that the death rate due to warfare is directly proportional to warfare frequency giving:

$$dN/dt = rN(1 - N/K) - NW \quad 4.10$$

If elevated warfare frequency causes each tribe to send out more war parties. The encounter rate leading to the initiation of new conflict, then, will be proportional to the product of population density and warfare intensity. This leads to the following equation describing the rate of change of W :

$$dW/dt = aNW - bW$$

4.11

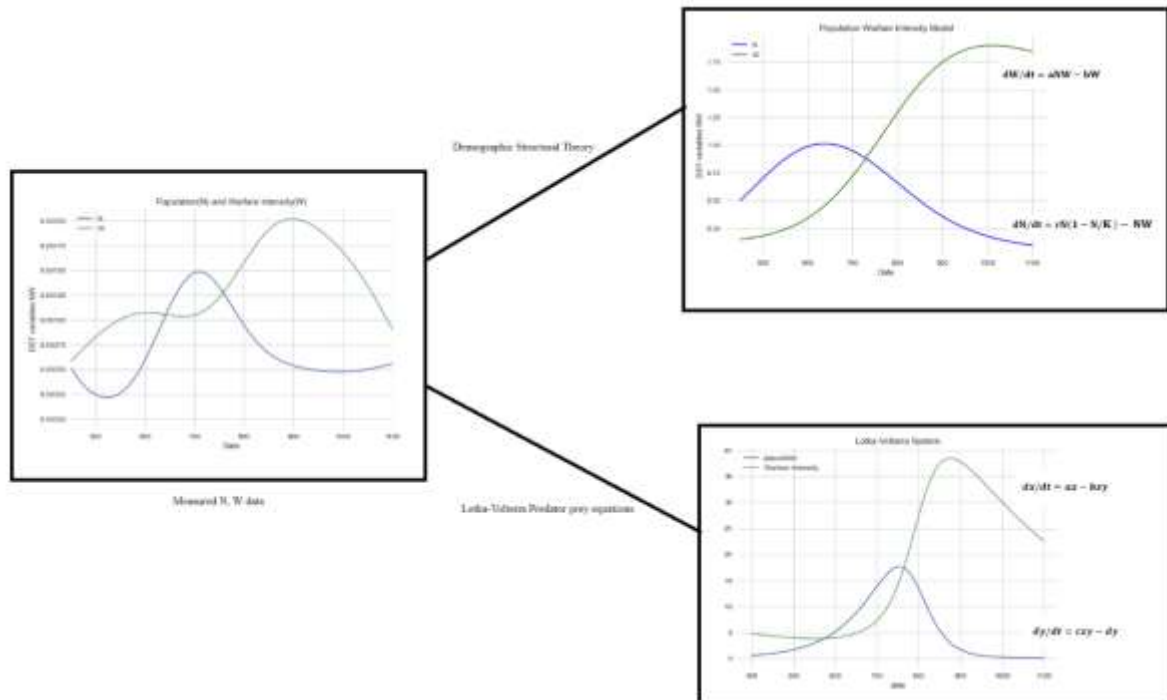


Figure 4-13 Cliodynamics Theory for NW

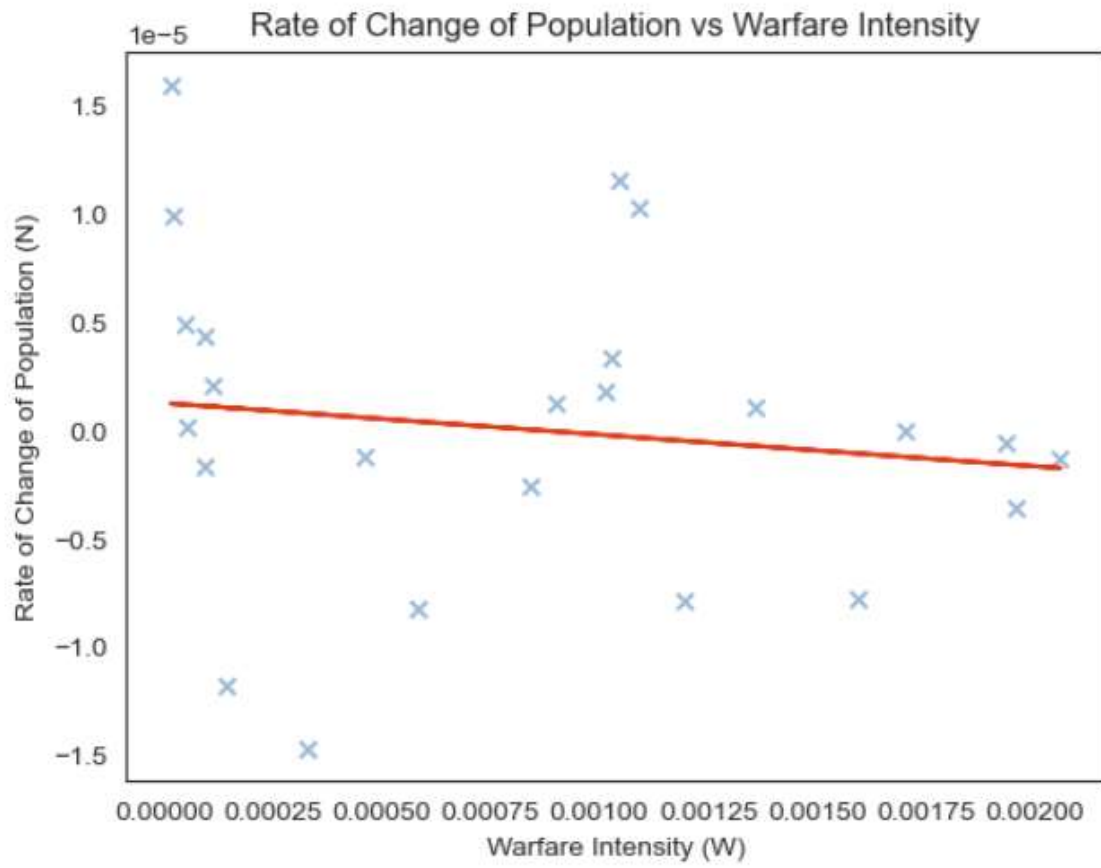


Figure 4-14 dN vs Warfare Intensity

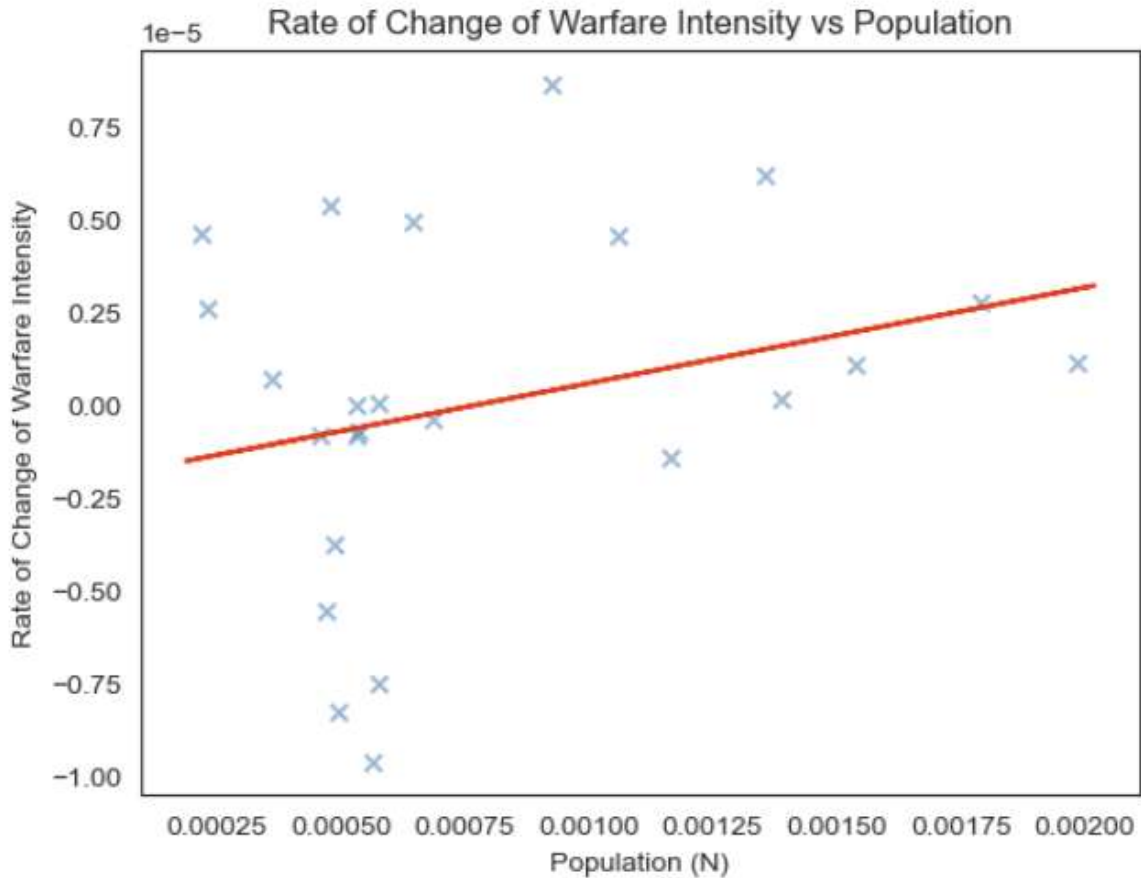


Figure 4-15 dW vs N

The population rate of change is negatively affected by warfare intensity as seen in fig. 4-13, while warfare rate of change is positively affected by population density as seen in fig. 4-14.

Sociopolitical Instability: Population, State Strength and Warfare Intensity

The dynamics of this model are illustrated in fig. 4-10. At the beginning of a cycle, both population (N) and state strength (S) grow. Increasing S suppresses internal warfare, with the

result that the carrying capacity $K(W)$ increases to the upper limit k_{max} . At the same time, the proportion of mortality due to warfare declines to zero. As a result, N increases rapidly. However, at a certain population size, the growth of state resources ceases, and S begins to collapse at an increasing rate, rapidly reaching 0. State collapse allows internal warfare to grow, and it rapidly reaches its maximum level. This means that $K(W)$ decreases, and mortality increases, leading to a population density collapse. Population decline eventually allows S to increase again, S suppresses W , and another cycle begins.

$$dN/dt = rN(1 - N/N_{max}) - \delta NW \quad 4.12$$

$$dS/dt = \rho N(1 - N/N_{max}) - \beta N \quad 4.13$$

$$dW/dt = aN^2 - \beta W - \alpha S \quad 4.14$$

where $N_{max} = k_{max} - cW$

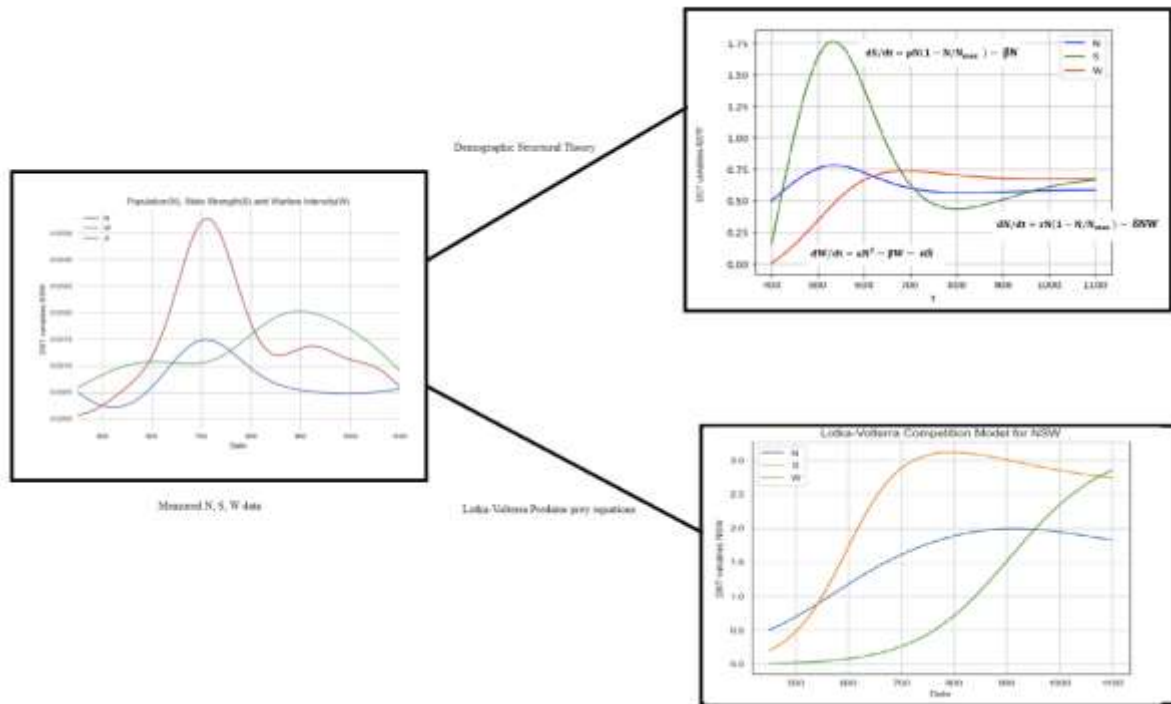


Figure 4-16 Cliodynamics Theory for NSW

The model makes specific predictions about the dynamic interaction between its variables. Define an index of sociopolitical instability as the difference between W and S . Thus, instability grows directly with internal warfare intensity and inversely with state strength. The general insight is that the interaction between population dynamics and sociopolitical instability results in coupled oscillations of the two variables.

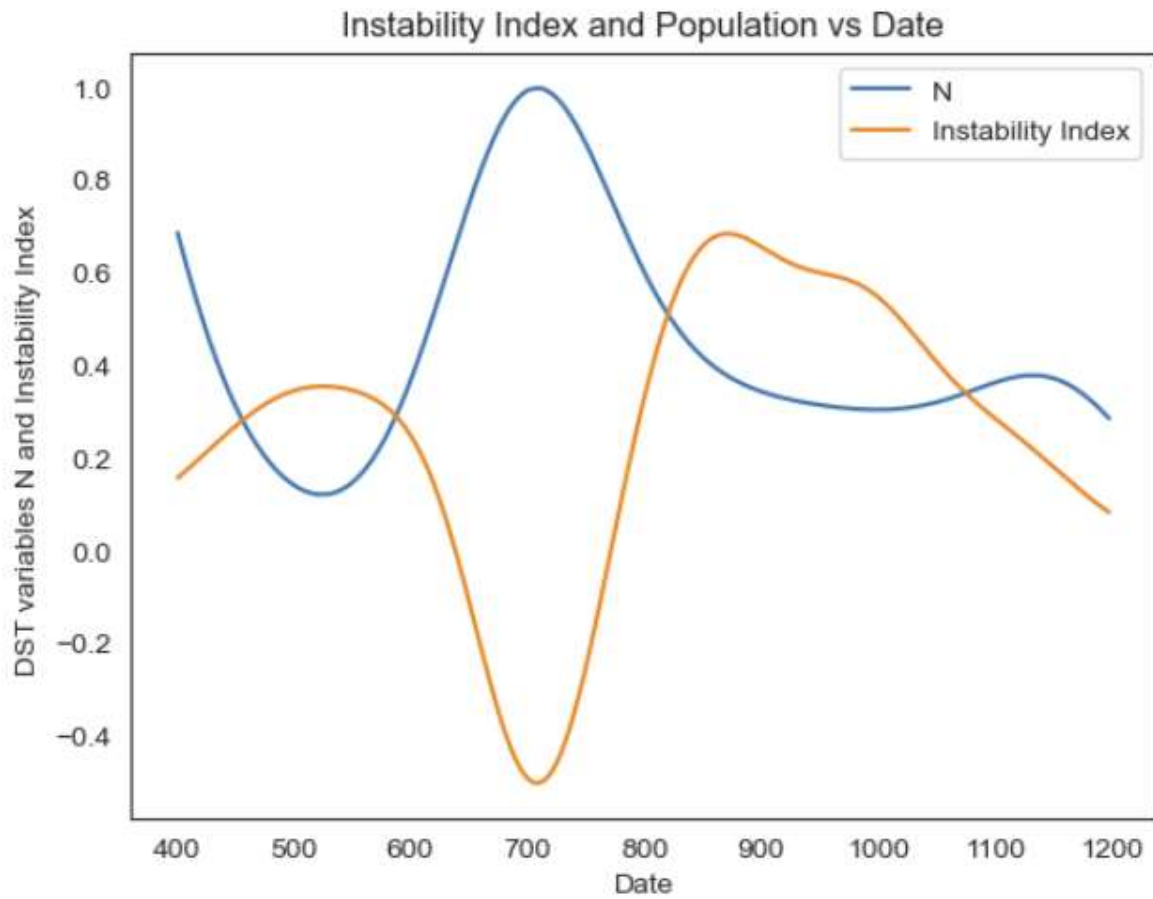


Figure 4-17 Measured Data for Population and Instability vs Date

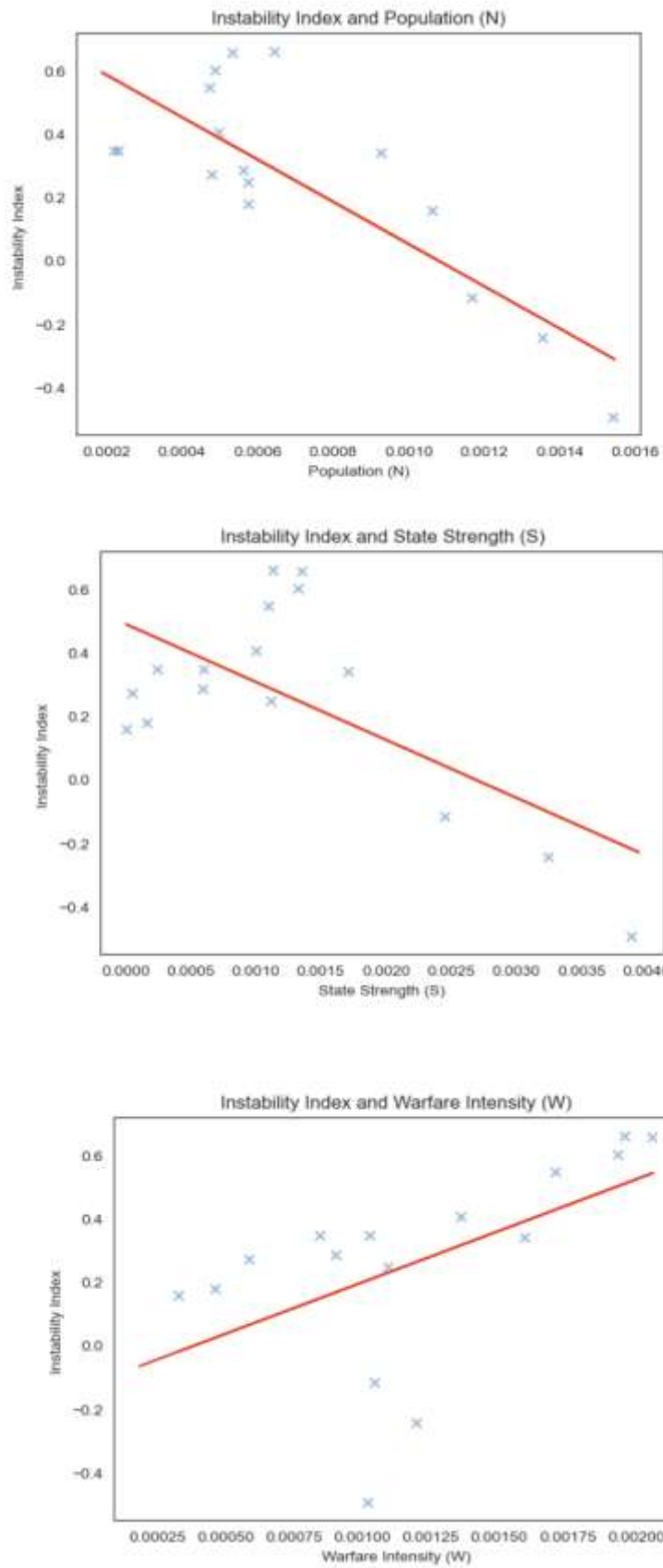


Figure 4-18 Instability and NSW Relationships

Cliodynamic Simulations

Developing a geopolitical spatial model of territorial dynamics

By characterising medieval agrarian states as an ecosystem of competing polities, Lotka-Volterra equations can analyse historical processes' dynamics. Initially, the variables and coefficients of the system of differential equations were derived from the concepts developed in geopolitical theory (Collins, 2010). In this model, the key variable is territory size, which varies depending on geopolitical resources such as taxes, military power, the logistic load of maintaining a territory, the costs of defence capability and resource extraction such as agriculture, and geographical features such as a coastal position or higher elevation.

$$dy/dt = cAexp(\sqrt{A}/h) - a \quad 4.15$$

A simple spatial model developed to study territorial evolution in Europe (Artzrouni & Komlos, 1996) can be used as a computational lens to examine the formation of polities within early medieval England. This spatial modelling approach can also be adapted within Cliodynamic computational simulations to model how warfare affects societies (Turchin, 2011) and (Turchin et al. 2013). The model assumes that each polity's military power function has the following mathematical form:

$$P_i(t) = A_i/(\alpha + exp(C\gamma_i + \beta)) \quad 4.16$$

A and C are the area and the perimeter of the polity, and α , β , γ are positive constants. In this model, the negative correlation of the logistic load parameter outweighs the positive correlation due to the increased territory, causing the magnitude of the polity's military power to decline to zero, stopping further territorial expansion gradually. One of the polities is chosen randomly to simulate a dynamic process, and its military strength is compared with the strength of all its

neighbours. The countries with the greatest power disparity began a war, which the militarily powerful country won, with some probability indicated by the formula:

$$\Pi = 1 - 0.5 \exp(-KP_i/(P_j - 1)) \quad 4.17$$

Where P_i and P_j are the power of the warring cells, and K is a parameter that tunes the deterministic character of the dynamics. A higher K -value denotes the higher possibility of the strongest country winning. The weakest country wins with a probability, $\Phi_i = 1 - \Phi_j$ (Kuperman, 2010). This model confirms that polities continue to expand until the logistic load stabilises their size. This spatial model and any model based purely on geopolitical mechanisms cannot wholly model the territorial dynamics. They cannot predict the exact dynamics of state growth and collapse because they have no mechanism for the fragmentation or collapse of states.

Initially, a shapefile of England and Wales was obtained, which described the geographical region under study. To run simulations, this region had to be divided into smaller cells. This was done using the QGIS vector processing tool to divide this shapefile into 10 km squares.



Figure 4-19 Shapefile with 10 km square grid.

The Artzrouni and Komlos model has five parameters (f , α , β , γ , and K). The investigators used the trial-and-error method to find the specific values of these parameters that would replicate Europe's historically observed dynamic map as closely as possible. In their paper (Artzrouni and Komlos, 1996), the values of the coefficients are given as $S = 15$, $\alpha = 1.00$, $\beta = 1.03$, $\gamma = 0.035$, and $K = 3.21$, and these have been used in this initial simulation.

Coastal position

Given that during the Middle Ages, state boundaries were not yet fully formed, we assume as a simplification that in 500 CE, England consisted of a mosaic of small square political units of side 10 km. Therefore, all units initially have the same area of 100 sq. km and are equally powerful. These cells, except those with a maritime border where the fraction f decreases their calculated perimeter and makes them slightly easier to defend. In their paper (Artzrouni and Komlos, 1996) the values of the coefficient f is given as 0.7.

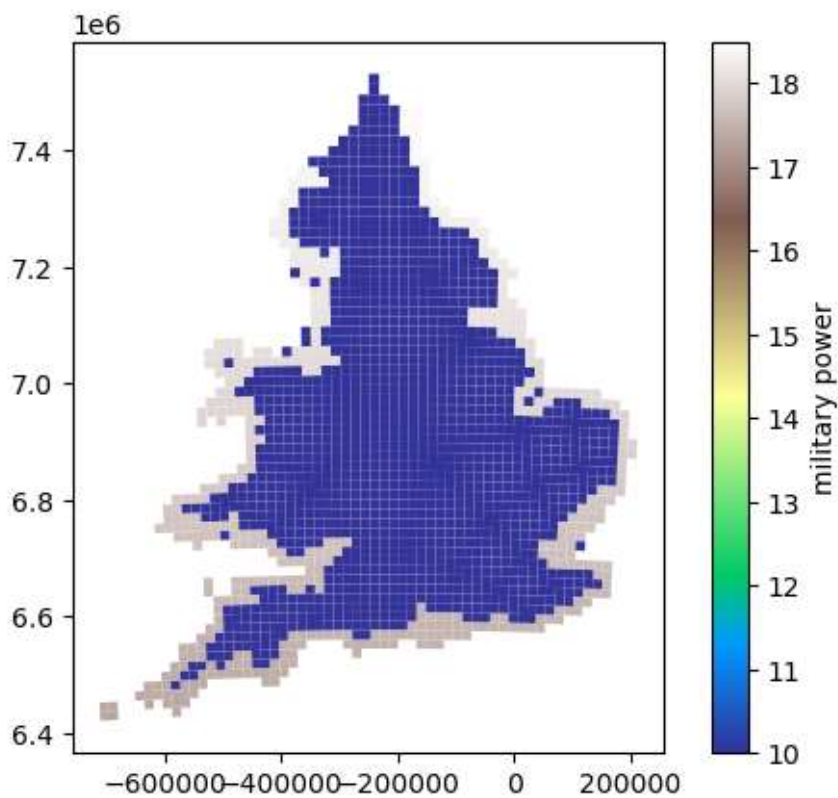


Figure 4-20 Effect of Coastal Position

The effect of elevation

Using the OS Terr50 geodatabase the elevation of hills and mountains can be determined using QGIS and used in the model. The effect of elevation is to increase military power when the cell is attacked (Turchin, et al., 2013). So that:

$$P_{def} = 1 + \gamma E_{def} \quad 4.18$$

where E_{def} is the elevation (in kilometres) of the defending cell found at spatial coordinates x and y , and γ is the coefficient translating elevation into defensive power (Turchin, et al., 2013)

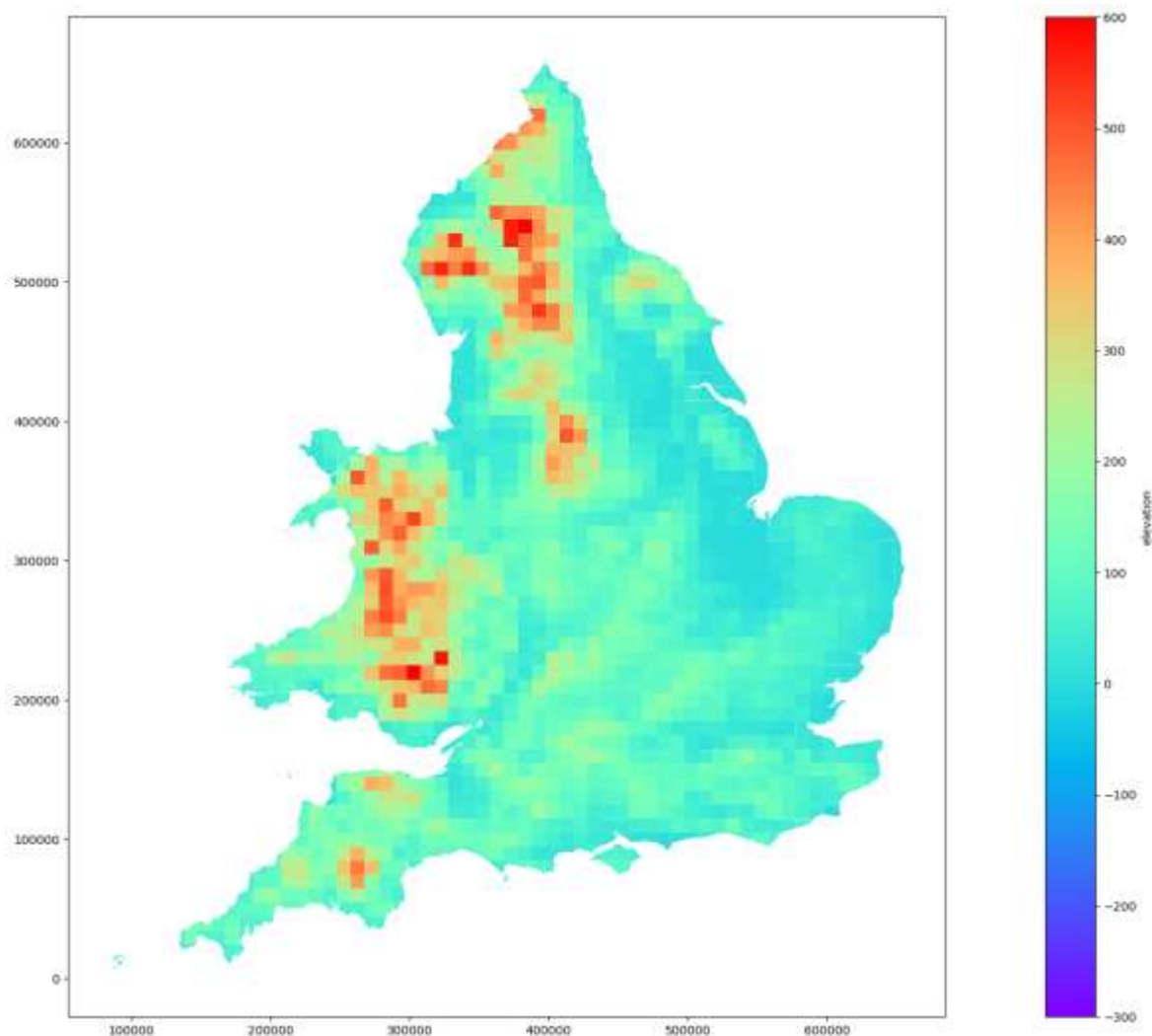


Figure 4-21 The Effect of Elevation

The effect of soil quality

Given that the communities at this time were agrarian and relied on agriculture, this factor was of some importance. The quality of the soils was derived from the Advanced Soil Geochemical Atlas of England and Wales, a joint project between the British Geological Survey and Rothamsted Research. The maps are based on 5700 surface soil samples (0-15cm) collected across England and Wales and analysed for 52 major and trace elements.

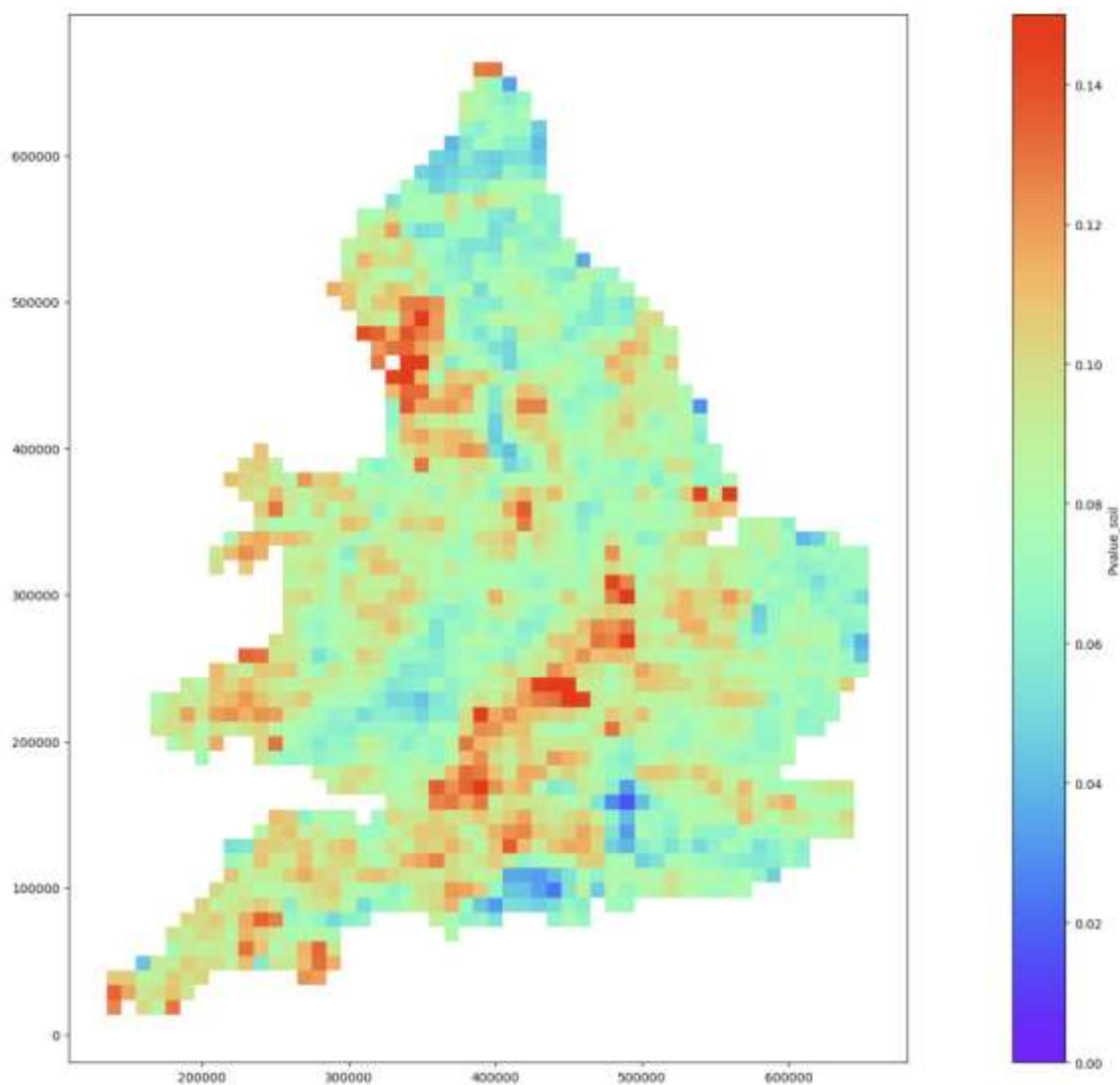


Figure 4-22 The Effect of Soil Quality Acknowledgement: This data is derived from research conducted by the British Geological Survey and Rothamsted Research and is based on soil samples collected for the National Soil Inventory by the Soil Survey of England and Wales, now the National Soil Resources Institute, Cranfield University.

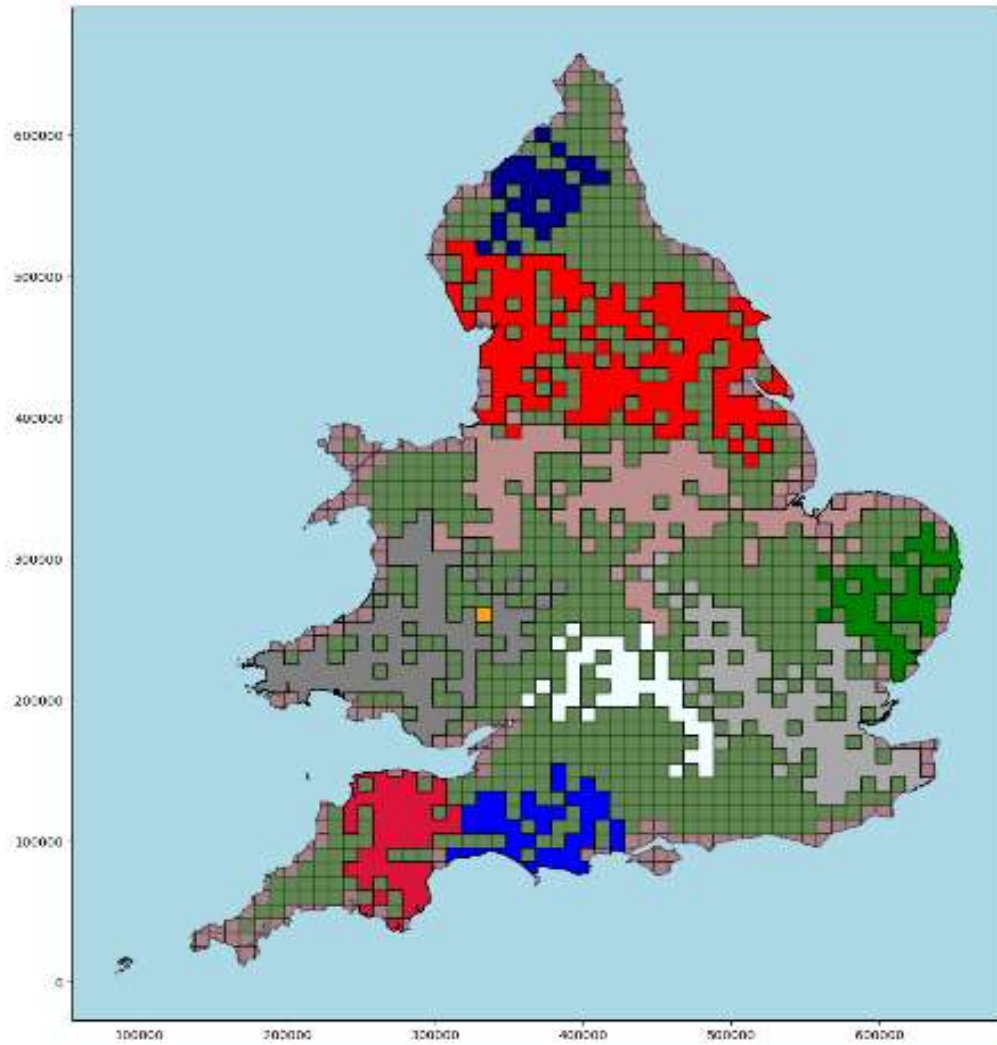


Figure 4-23 Mid Run of Geopolitical Simulation

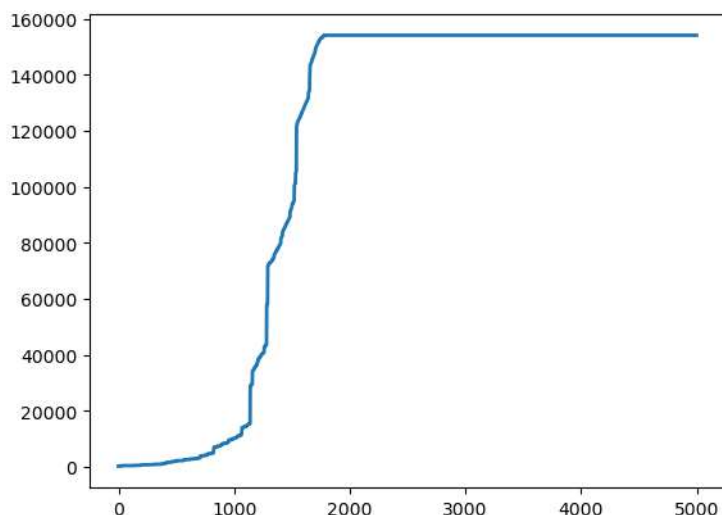


Figure 4-24 Area vs time for polity in geopolitical theory

The Metaethnic Frontier Model

The next refinement is to account for the social cohesion of the community. According to the metaethnic frontier model of territorial dynamics, each cell has a level of social cohesion or Asabiyya S , which varies with the cell's spatial position relative to the polity's centre. Turchin (2003) suggests a logistic growth model for the Asabiyya such that:

$$A' = C_0AS(1 - A/h) - a \quad 4.19$$

and

$$S' = r_0(1 - A/2b)S(1 - S) \quad 4.20$$

Turchin (2003) uses the parameters: $r_0 = 0.1$, $c_0 = 1$, $h = 1$ and $a = 0.1$ which generates a dynamic system undertaking oscillatory behaviour.

The theory proposes that agrarian empires rise in regions near fault lines between religions, ethnicities, or cultures. In such areas, the challenging conditions necessitate growth in the capacity for collective action called Asabiyya, in the parlance of Turchin (2003). When an empire grows to a size where such borderlands are small compared to the overall area, the

conditions for collective action dwindle, and Asabiyya declines, potentially causing an imperial collapse.

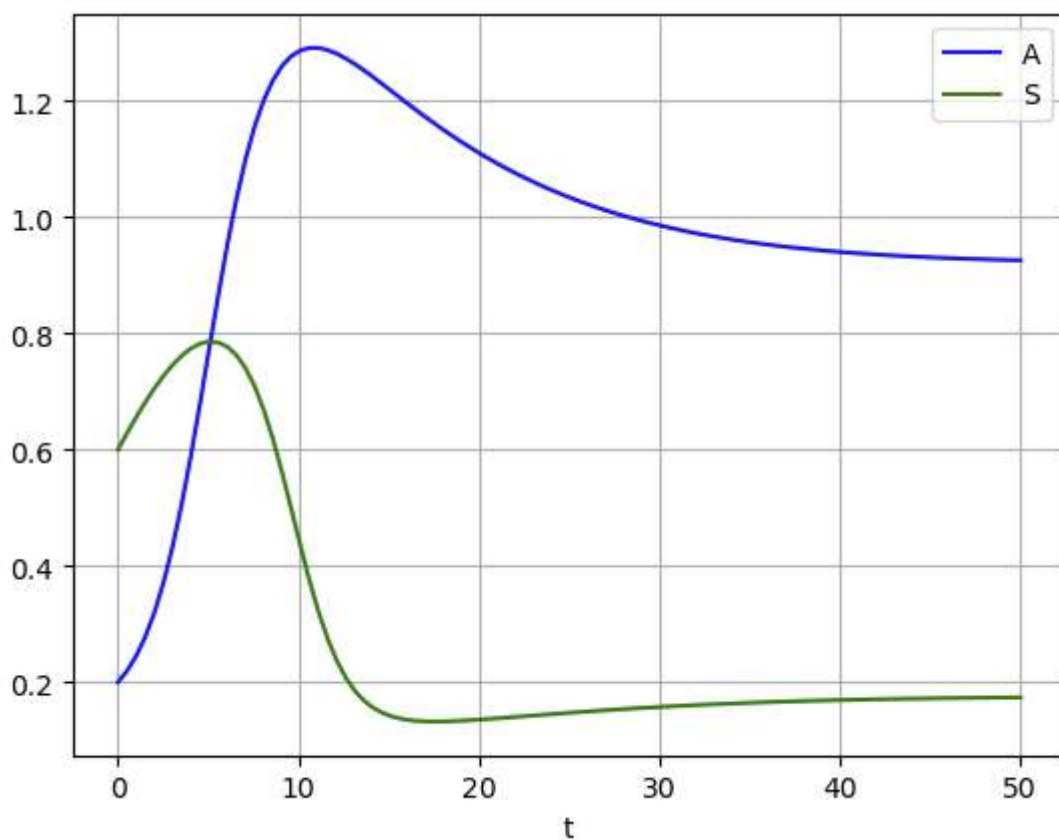


Figure 4-25 Variation of A and S

Metaethnic Frontier Theory Simulation

The relationship is easy to model in a simulation. It has the interesting feature of producing oscillations in empire size, i.e. rises and falls, as well as long periods of stability followed by periods of rapid territorial change. Historical records support this pattern, and it emerges from a very small set of rules. Here is how it works:

We start with an arena of cells, each with an "imperial index" and an Asabiya value (S_x). The arena is seeded with one polity made of multiple cells, each with an Asabiya of 1 and an

imperial index of 1 (a seed empire). All other cells are initiated with an Asabiya and imperial index of 0 and assigned to the 'hinterlands of very small non-state polities. At each time step, the Asabiya of each cell on the metaethnic frontier, where it borders another cell with a different imperial index, is increased by:

$$S = r_0(S(1 - S)) \quad 4.21$$

where r_0 is a constant that can be tweaked in the simulation. However, if the cell is not on a border the Asabiyya of each cell is decreased by:

$$S = S - \delta S \quad 4.22$$

Where again, δ is a tweakable constant.

Then, each cell considers an attack on its neighbours with a military power P equal to the following equation where A is the number of other cells that share its imperial index (empire size), \underline{S} is the averaged Asabiya of all cells that share its imperial index (empire Asabiyya), d_x is the distance of the cell to the centroid of all other cells that share its imperial index (distance to the capital), and h is a special constant:

$$P_x = A_x \underline{S} \exp(-d_x/h) \quad 4.23$$

The defending cell's power is computed the same way. If the difference between the attacking and defending cell's power is greater than the threshold Δ , then the attacking empire annexes the cell (switches imperial index). The Asabiya is clamped between 0 and 1, and if, at any step, the Asabiya of a polity drops below a constant. *Scrit* then the polity is destroyed (each cell is re-initialised with index 0).

One of the most critical parameters is the constant h . This constant defines how an empire's power declines with distance from its centre. The intriguing oscillations and alternating periods of stability and chaos only occur if the h value is in a certain range. Too small, and empires cannot rise; too big, and they expand until they collide and stay in equilibrium.

Social Cohesion

Without a critical Asabiyya S_{crit} , the seed polity expands to fill the available territory completely.

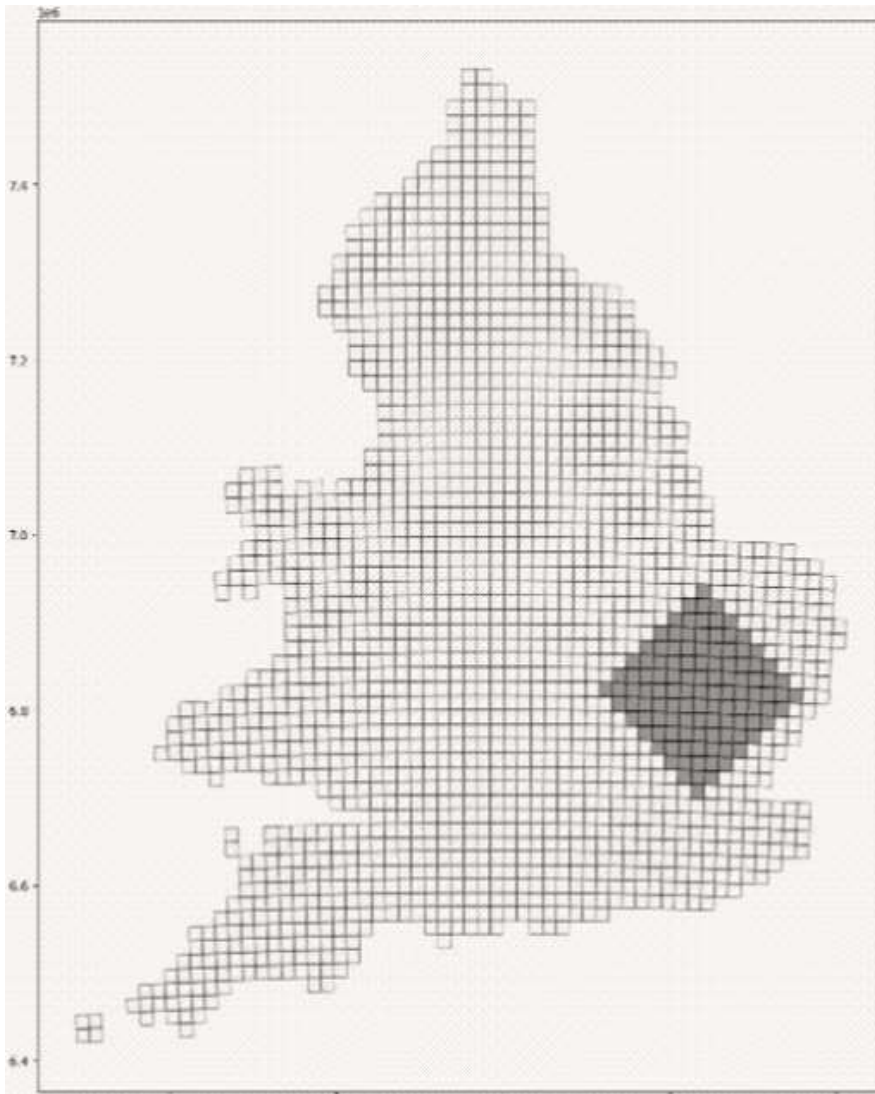


Figure 4-26 Effect of Critical Asabiyya

Conversely, when the value of the critical asabiyya S_{crit} is ~ 0.01 , the polity can only reach a limited size until it breaks down into fragmentary communities of the hinterlands again.

When the metaethnic frontier theory is completely implemented, the polities expand to a moderate size, and there is a process of continuous formation and disintegration along the frontiers.

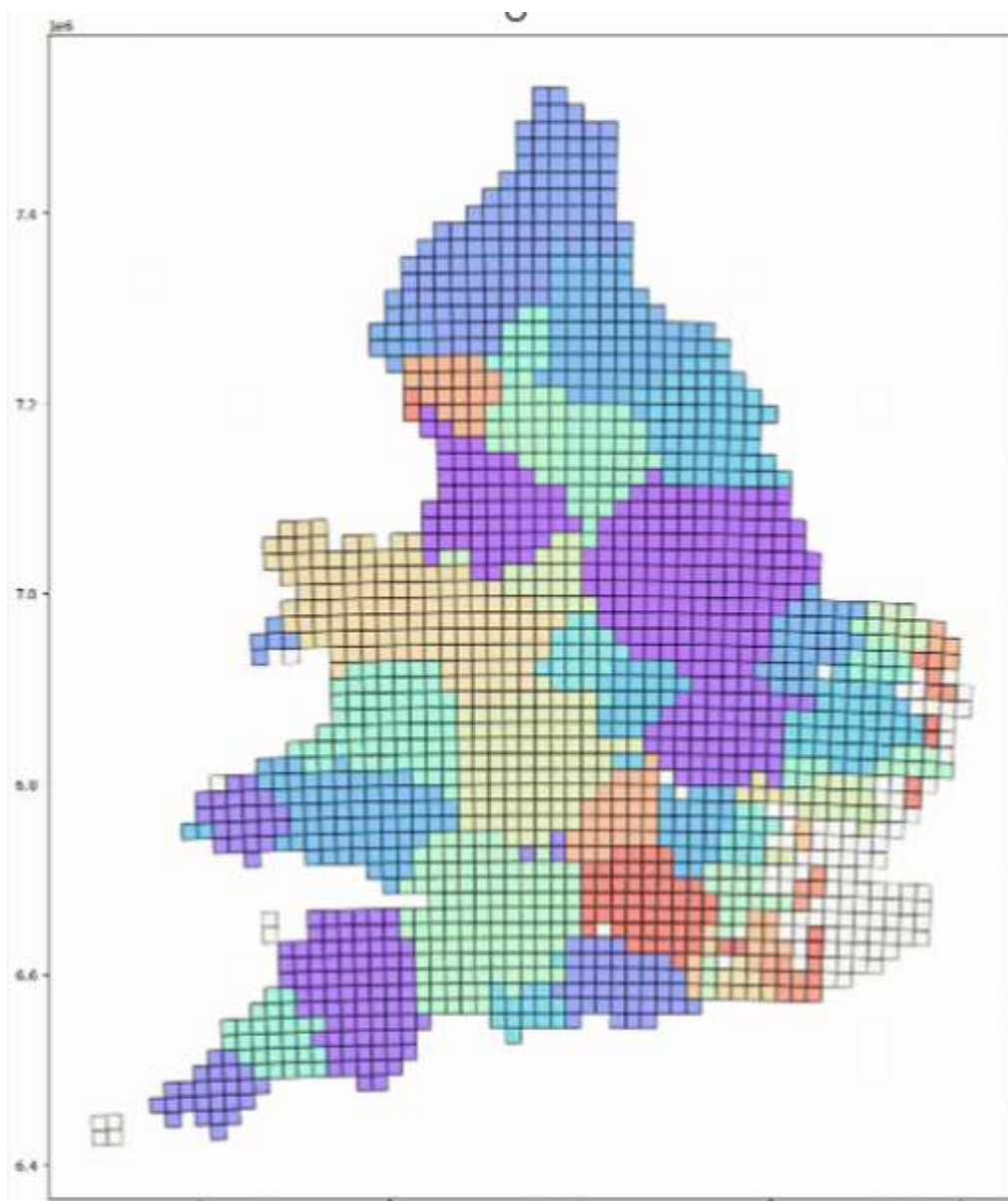


Figure 4-24 Implementation of the Metaethnic Frontier Theory

Diffusion Processes

Migration or invasion can be simulated using an optimised cost function to calculate the least energy route. Only the terrain elevation is initially considered, but preexisting population, waterways and agricultural land quality can also be considered. Here, the simulation models the spread of migrants arriving on the shores of East Anglia.

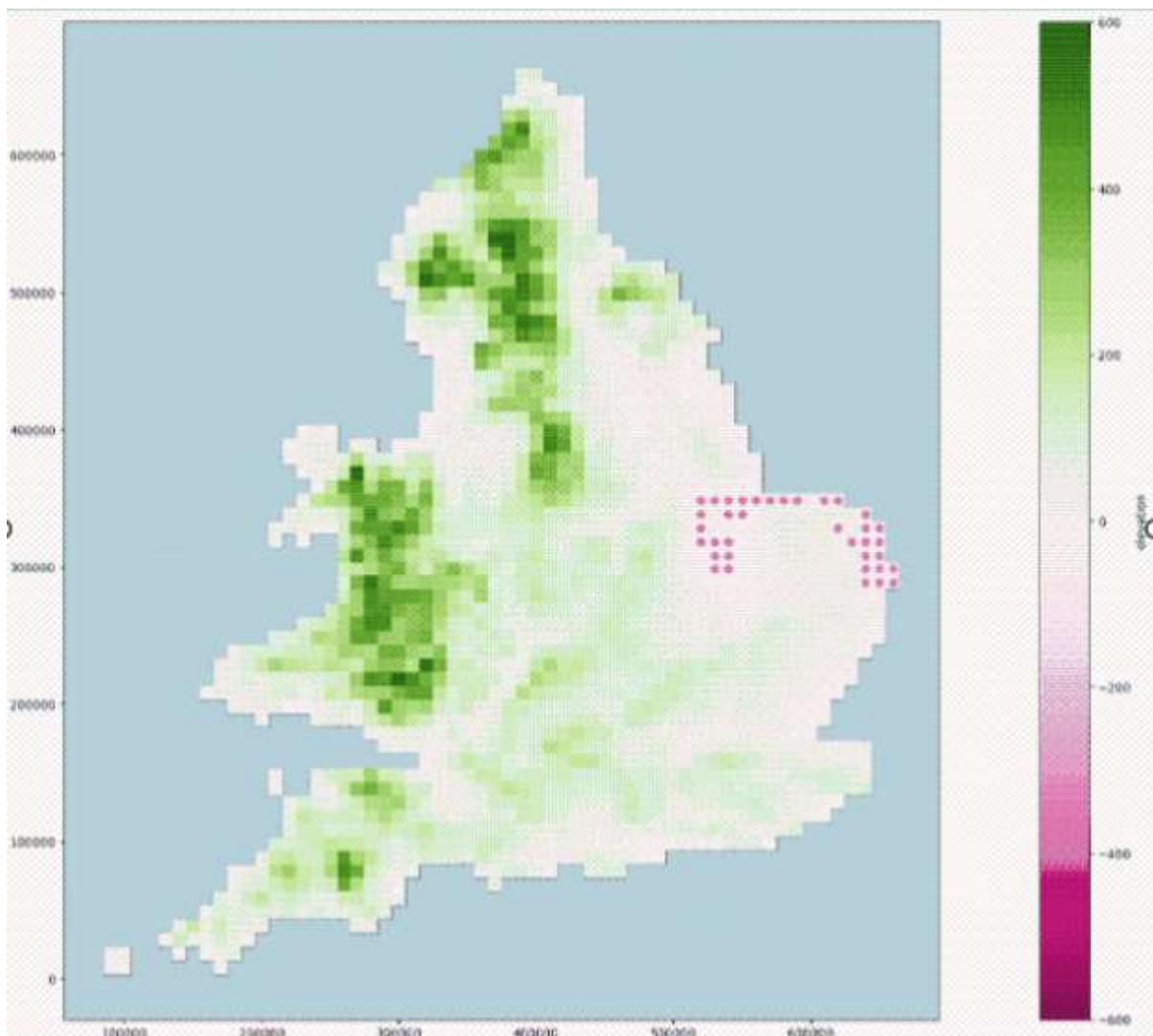


Figure 4-25 Diffusion of Settlers

Chapter 5 Discussion

This research aimed to explain the temporal and spatial distribution of early medieval kingdoms in England using insights derived from Cliodynamics. Radiocarbon data was used to discover the temporal and spatial variation of activity from the early medieval period in England, as shown in Figure 3-1 and Figure 3-3. Archaeological data were used, as shown in Figure 3-4 and Figure 3-7, to discover the temporal and spatial variation of artefacts from the early medieval period in England. Numismatic analysis of coins was used to estimate the temporal and spatial variation of the influence of Anglo-Saxon kingdoms in early medieval England, as shown in Figure 3-12 and Figure 3-13.

The key Cliodynamic variables were identified and then measured using appropriate proxy variables from archaeological and historical evidence. These results demonstrated the existence of secular cycles within these variables and relationships between the variables, which were explicable in terms of the Demographic Structural Theory and its derived models. The Demographic Fiscal model connects Population (N) and State Strength (S), the War model connects Population (N) and Warfare Intensity (S) and sociopolitical instability connects Population (N), State Strength (S), Warfare Intensity (S) as seen in Figure 4 – 8, Figure 4 – 9 and Figure 4 - 10.

The empirical results support the population-warfare theory: there is a tendency for population numbers and internal warfare intensity to oscillate at the same period, but with a phase shift, the warfare peaks following the population peaks, as seen in Figure 4-8. Furthermore, the rates of change of the two variables behave precisely as predicted by the theory: the population rate of change is negatively affected by warfare intensity, as seen in Figure 4-14, while the warfare rate of change is positively affected by population density, as seen in Figure 4-15.

In the sociopolitical instability model, Figures 4-17 and 4-18 show that the instability index grows directly with internal warfare intensity and inversely with state strength, confirming the general insight that the interaction between population dynamics and sociopolitical instability results in coupled oscillations of the two variables.

The population in Early Medieval England is seen in Figure 4-10, which shows an increasing population from 550 CE to 700 CE, followed by a decline from 700 CE to 1066 CE, which implies a secular cycle from 550 CE to 1066 CE. However, Alexander (2016) posits that an Anglo-Saxon secular cycle occurred from 878 CE to 1066 CE.

The analysis of Anglo-Saxon coins in Figure 4-6 indicates that Alexander's (2016) analysis applies to a secular cycle of the rise and fall of the House of Wessex rather than Anglo-Saxon society. This occurs because Alexander (2016) used the area of the Wessex kingdom as a proxy for the population and therefore, did not analyse the prior cycle involving the Mercian House, the Northumbrian and the Angles. However, a prior secular cycle from the sixth to the ninth centuries was briefly mentioned in the paper.

This thesis indicates that there were two secular cycles during the Anglo-Saxon period. The first cycle, 550 – 876 CE, was dominated by Mercia and ended with the Viking invasion of England. The second cycle, 878 – 1066 CE, was dominated by Wessex and ended with the Norman conquest in 1066 CE. These two secular cycles fit within the secular cycle describing Anglo-Saxon England.

This is a general finding, as the numismatic analysis of Anglo-Saxon coins in Figure 4-7 illustrates that kingdoms or polities frequently rose and fell throughout the Anglo-Saxon

period. This point is also demonstrated within the metaethnic frontier theory simulation, where polities expand and contract over time.

The spatial distribution of the kingdoms is influenced by the terrain and vegetation, as seen in Fig. 3-24 and in the simulations showing the effects of soil, elevation, and coastal position. The geopolitical simulation explains the formation of larger kingdoms through warfare and conquest, from the fragmentary holding of many local tribes to the formation of several larger kingdoms and finally to a unified English state. The metaethnic frontier theory allows for the breakup of polities and the continual process of formation and fragmentation seen historically. The area of the polities depends on the value of social cohesion, with higher degrees of social cohesion supporting larger kingdom areas.

John Blair (2000) suggests that shared threats and external conflicts, such as Viking raids and invasions, could have united the Anglo-Saxons against a common enemy. This would have facilitated a shared purpose and identity, leading to greater social cohesion and, eventually, a unified English state.

Strong leadership and governance could have increased social cohesion during the Anglo-Saxon period in England. A well-functioning legal system that ensured justice and stability would have contributed to social cohesion by minimising internal conflict and promoting trust. Cultural and religious factors, such as developing a common language (Old English) and shared cultural practices, would help foster a sense of collective identity and belonging. The spread of Christianity provided a unifying religious framework and moral code, promoting shared values and beliefs. Economic and social factors, such as agriculture and trade with sufficient food production and thriving trade networks, would contribute to social stability and contentment.

The Anglo-Saxon period lasted for several centuries, during which social cohesion varied across different regions and over time. The relative importance of these factors depended on specific historical circumstances. While archaeological and historical evidence offers some insights into Anglo-Saxon society, understanding the complexities of social cohesion remains challenging. However, by considering these various factors, we can better appreciate the dynamics that shaped social cohesion during this formative period in English history.

While cliodynamics provides a robust framework for quantitatively analysing historical processes, it has challenges. The complexity and uniqueness of historical events do not always lend themselves to purely quantitative modelling and traditional historical analysis; focusing on narrative and context remains important for understanding the nuances and complexities of historical events. A balanced approach that integrates narrative and qualitative methodologies offers a more comprehensive understanding of history. Furthermore, whilst relying on empirical regularities and generalisations can obscure the unique, context-specific factors that shape historical events, cliodynamics offers valuable insights by identifying historical patterns and causal mechanisms.

Chapter 6 Conclusions

Cliodynamics, a new field, offers a quantitative and scientific approach to understanding historical processes, contrasting with traditional historical analysis, which often relies on qualitative narratives and interpretative methods. This comparison highlights these approaches' methodological differences and potential complementarities, emphasising how cliodynamics can enhance historical analysis through its predictive and analytical capabilities.

Cliodynamics, as an interdisciplinary approach to history by applying mathematical modelling and statistical analysis to historical data, cliodynamics introduces quantitative rigour that can complement traditional historical narratives. It allows for testing hypotheses and identifying patterns and trends that might not be readily apparent through qualitative analysis alone. Cliodynamics often focuses on long-term historical processes spanning centuries or even millennia. This enables the identification of large-scale patterns and cycles that a focus on shorter timeframes might obscure.

Cliodynamics draws on various disciplines, including history, mathematics, sociology, anthropology, and ecology. This encourages collaboration between scholars from different fields, leading to new insights and perspectives. While historical prediction is inherently complex and uncertain, cliodynamics offers the potential to develop models that can identify future trends and scenarios based on past patterns. Cliodynamics can help to generate new research questions and approaches to historical study, pushing the boundaries of traditional historical inquiry.

There is a strong relationship between archaeology and cliodynamics, and this research has integrated archaeological data with quantitative and predictive cliodynamics methodologies. This interdisciplinary approach enhances our understanding of historical processes by combining the empirical richness of archaeology with the analytical rigour of cliodynamics. The integration of these fields could offer a comprehensive framework for examining cultural and societal changes over time. Overall, cliodynamics offers a valuable and complementary approach to historical study. Combining the rigour of quantitative analysis with the insights of multiple disciplines can provide a deeper and more nuanced understanding of historical processes and patterns.

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